Negotiation: Strategic Approach

(September 3, 2007)

How to divide a pie / find a compromise among several possible allocations?

☞ Wage negotiations

☞ Price negotiation between a seller and a buyer

Bargaining Situation:

1/ (i) Individuals are able to make *mutually beneficial agreements*

(ii) There is a *conflict of interest* over the set of possible agreements

(iii) Every agent can individually reject any proposal

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Before Nash (1950, 1953), the only solution proposed by economic theory is that the agreement should be:

- individually rational (i.e., better than full disagreement)
- Pareto optimal (i.e., no other agreement is strictly better for all agents)

Nash suggests two kinds of solutions:

❶ The *axiomatic approach*: what properties should the solution satisfy?

❷ The *strategic* (non-cooperative) *approach*: what is the equilibrium outcome of a specific and explicit bargaining situation?

Here, strategic approach ❷: The bargaining problem is represented as an extensive form game (alternating offers, perfect information)

➥ Explicit bargaining rules

Two players bargain to share an homogeneous "pie" (surplus), whose size is normalized to 1

An *offer* is a pair (x_1, x_2)

Set of all possible *agreements* (Pareto optimal offers):

$$X = \{(x_1, x_2) \in \mathbb{R}^2_+ : x_1 + x_2 = 1\}$$

Examples:

➢ Sharing one euro: $x_i =$ amount of money for player i

➢ Price negotiation: $x_2 =$ price paid by the buyer (player 1) to the seller (player 2)

➢ Wage negotiation: $x_1 =$ profit of the firm (player 1)

Preferences: Player i prefers $x = (x_1, x_2) \in X$ to $y = (y_1, y_2) \in X$ iff $x_i > y_i$

Point of Departure: Ultimatum Game (continuous)

First period: player 1 offers $x = (x_1, x_2) \in X$

Second period: player 2 Accepts (A) or Rejects (R) the offer. If he rejects they both get 0

Extensive form:

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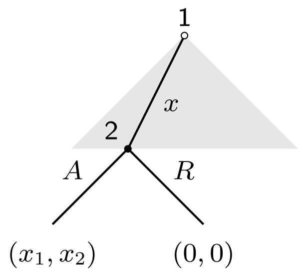

Is every agreement a Nash equilibrium outcome?

Unique SPNE: player 1 proposes $(1, 0)$ and player 2 accepts every offer

But the ultimatum game is usually not appropriate because **player 1 has all the bargaining power**

Assume that player 2 can make a counter offer, that player 1 should accept or reject

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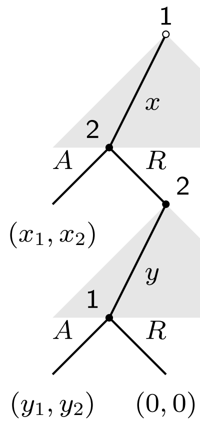

Now, it is player 2 who has all the bargaining power

Backward induction $\Rightarrow$ solution $y = (0, 1)$ and A in the second period

$\Rightarrow x = (0, 1)$, or $x \neq (0, 1)$ and R in the first period

$\Rightarrow$ at every SPNE player 2 obtains all the pie

More generally, whatever the length of the game, the player who makes the last offer obtains all the pie

But time is valuable, delay in bargaining is costly ...

Discount factor $\delta_i \in (0, 1)$ for player i

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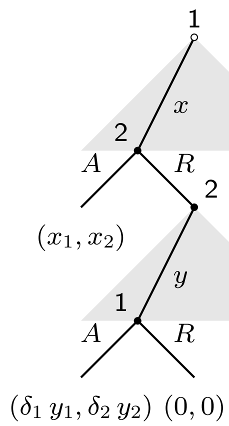

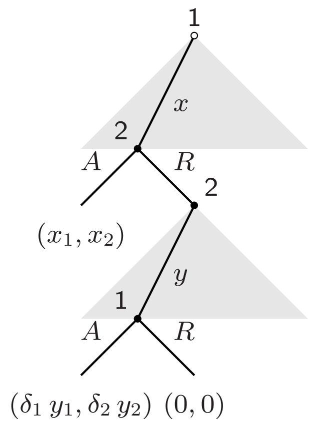

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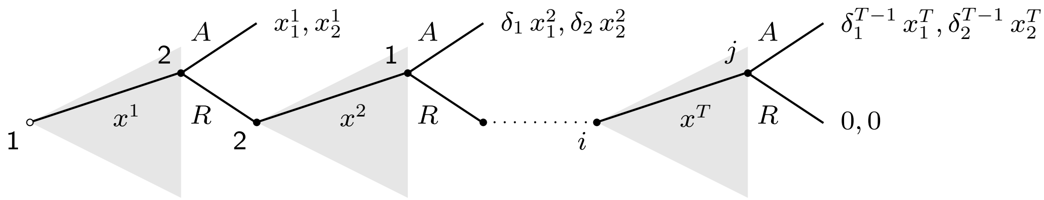
Backward induction:

Subgame after player 2's rejection: unique SPNE: player 2 proposes $(0, 1)$ and player 1 accepts every offer $\Rightarrow$ payoff $(0, \delta_2)$

Subgame after player 1's proposal: player 2 accepts $x_2 \geq \delta_2$ and rejects $x_2 < \delta_2 \Rightarrow$ player 1 proposes $(x_1, x_2) = (1 - \delta_2, \delta_2)$ in the first period

Finite Horizon Bargaining



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i (j) = player 1 if T is odd (even) $\quad$ i (j) = player 2 if T is even (odd)

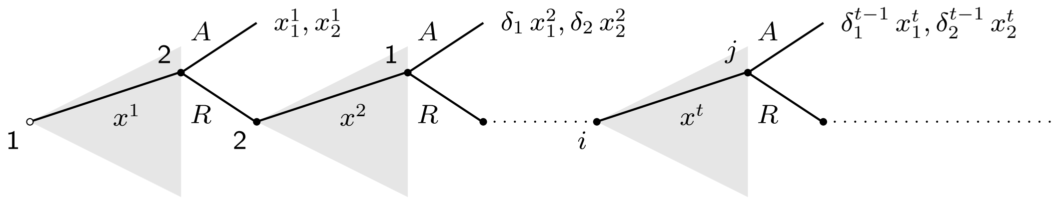
➥ Backward induction

✍ Check that if $T = 3$ then $x^1 = (1 - \delta_2(1 - \delta_1), \delta_2(1 - \delta_1))$

✍ Check that if $T = 4$ then $x^1 = (1 - \delta_2(1 - \delta_1(1 - \delta_2)), \delta_2(1 - \delta_1(1 - \delta_2)))$

Problem: the solution depends significantly on the exact deadline

Infinite Horizon Bargaining



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i (j) = player 1 if t is odd (even)

i (j) = player 2 if t is even (odd)

Remarks.

- Every subgame starting with player 1's offer is equivalent to the entire game
- Unique asymmetry in the game tree: player 1 is the first to make an offer
- It is common knowledge that players only care about the final agreement x and the period at which this agreement is reached (very strong assumption)
- The structure of the game is repeated, but it is not a repeated game ($A \Rightarrow$ end of the "repetition")

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Pure strategy of player 1: Sequence $\sigma = (\sigma^t)_{t=1}^{\infty}$, where

$$\sigma^t : X^{t-1} \to X \quad \text{if } t \text{ is odd}$$
$$\sigma^t : X^{t-1} \to \{A, R\} \quad \text{if } t \text{ is even}$$

Pure strategy of player 1: Sequence $\tau = (\tau^t)_{t=1}^{\infty}$, where

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$$\tau^t : X^{t-1} \to X \quad \text{if } t \text{ is even}$$
$$\tau^t : X^{t-1} \to \{A, R\} \quad \text{if } t \text{ is odd}$$

Stationary strategies: do not depend on the period and on past offers

Player 1:

$$\sigma^t(x^{t-1}) = x^* \qquad \text{if } t \text{ is odd}$$
$$\sigma^t(x^{t-1}) = \begin{cases} A & \text{if } x_1^{t-1} \geq \overline{x}_1 \\ R & \text{if } x_1^{t-1} < \overline{x}_1 \end{cases} \qquad \text{if } t \text{ is even}$$

Player 2:

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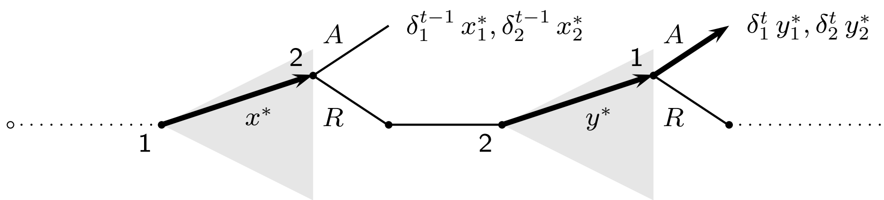
$$\tau^t(x^{t-1}) = y^* \qquad \text{if } t \text{ is even}$$
$$\tau^t(x^{t-1}) = \begin{cases} A & \text{if } x_2^{t-1} \geq \overline{y}_2 \\ R & \text{if } x_2^{t-1} < \overline{y}_2 \end{cases} \qquad \text{if } t \text{ is odd}$$

Accepted offers at the SPNE: $\forall\, t$, $\forall\, \delta < 1$ ➠ $y_1^* = \overline{x}_1$ and $x_2^* = \overline{y}_2$

Player 2 in (odd) period t given those strategies:



15/ Equilibrium $\Rightarrow$ $\delta_2^{t-1}\, x_2^* = \delta_2^t\, y_2^*$, i.e., $x_2^* = \delta_2\, y_2^*$

Symmetric reasoning for player 1 $\Rightarrow$ $y_1^* = \delta_1\, x_1^*$

Hence

$$x^* = \left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right)$$
$$y^* = \left(\frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}, \frac{1-\delta_1}{1-\delta_1\delta_2}\right)$$

✍ Find a Nash equilibrium (specify the complete strategies, the outcome and the payoffs) that is not Pareto optimal. Explain why this Nash equilibrium is not a subgame perfect Nash equilibrium

Proposition. (Rubinstein, 1982) *The preceding stationary strategy profile, i.e.,*

- *Player 1 always offers x^* and accepts an offer x iff $x_1 \geq y_1^*$*
- *Player 2 always offers y^* and accepts an offer x iff $x_2 \geq x_2^*$*

16/ *where*

$$x^* = \left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right)$$
$$y^* = \left(\frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}, \frac{1-\delta_1}{1-\delta_1\delta_2}\right)$$

is the unique subgame perfect Nash equilibrium of the alternating offer bargaining game with perfect information

Equilibrium Properties.

- *Efficiency* in the sense of Pareto (no delay)
- *Patience* of player i increases ($\delta_i \uparrow$) $\Rightarrow$ player i's share increases
- *First-mover advantage*: if $\delta_1 = \delta_2$ the first player gets $\frac{1}{1+\delta} > \frac{1}{2}$, but $\frac{1}{1+\delta} \longrightarrow \frac{1}{2}$ as $\delta \to 1$

17/ **Remarks.**

➢ If proposals are simultaneous in each period then every Pareto optimal share is a SPNE outcome

➢ If only one player is able to make offers then, at a SPNE, he obtains all the pie in the first period

Risk of Breakdown

After every rejection, negotiations terminate with probability $\alpha \in (0, 1)$

$\Rightarrow$ Even if players are very patient (assume $\delta_1 = \delta_2 = 1$) there is a pressure to agree rapidly

Payoffs when negotiations terminate: $(b_1, b_2) \in \mathbb{R}^2_+$, with $b_1 + b_2 < 1$

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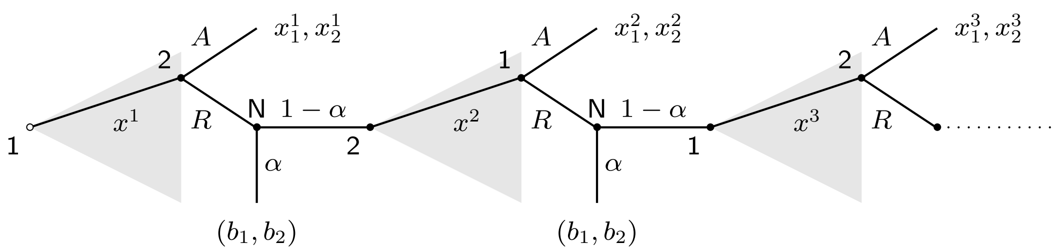

As in the basic model the unique SPNE is a stationary strategy profile

- Player 1 always proposes x^* and accepts a proposal x iff $x_1 \geq y_1^*$
- Player 2 always proposes y^* and accepts a proposal x iff $x_2 \geq x_2^*$

Player 1 at some period given this strategy:

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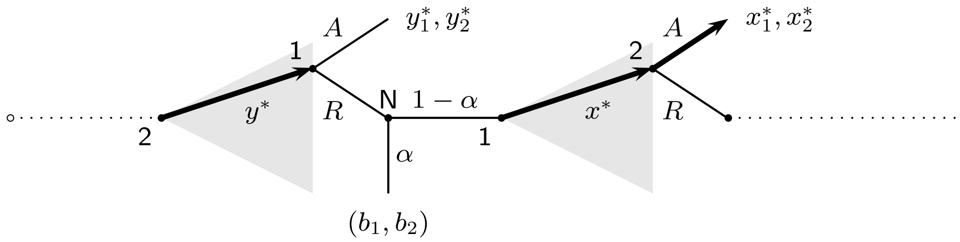

Equilibrium $\Rightarrow$ $y_1^* = \alpha\, b_1 + (1-\alpha)\, x_1^*$

Symmetric reasoning for player 2 $\Rightarrow$ $x_2^* = \alpha\, b_2 + (1-\alpha)\, y_2^*$

Hence

$$x^* = \left(\frac{1 - b_2 + (1-\alpha)\, b_1}{2-\alpha}, \frac{(1-\alpha)(1-b_1) + b_2}{2-\alpha}\right)$$

$$y^* = \left(\frac{(1-\alpha)(1-b_2) + b_1}{2-\alpha}, \frac{1 - b_1 + (1-\alpha)\, b_2}{2-\alpha}\right)$$

Allocation when the probability of breakdown $\alpha \longrightarrow 0$:

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$$x^* \longrightarrow \left(b_1 + \frac{1-b_1-b_2}{2}, b_2 + \frac{1-b_1-b_2}{2}\right)$$

➥ Each player gets his payoff in the event of breakdown (b_i) and we split equally the excess of the pie ($\frac{1-b_1-b_2}{2}$)

References

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——— (1953): "Two Person Cooperative Games," *Econometrica*, 21, 128–140.

Rubinstein, A. (1982): "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50, 97–109.