Negotiation: Strategic Approach

(September 3, 2007)

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Bargaining Situation:

(i) Individuals are able to make mutually beneficial agreements

(ii) There is a conflict of interest over the set of possible agreements

(iii) Every agent can individually reject any proposal


WHAT DO YOU THINK IS THE BEST WAY TO GET WHAT YOU WANT? IS IT BETTER TO HOLD FAST AND NEVER BACK DOWN, OR TO COMPROMISE?
I SUPPOSE IT'S BEST TO HOLD FAST WHEN YOU CAN, AND COMPROMISE WHEN YOU NEED TO.
THAT'S A LOT MORE MATURE THAN I THINK I CARE TO BE.

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➥ Explicit bargaining rules

Two players bargain to share an homogeneous “pie” (surplus), whose size is normalized to 1

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Preferences: Player i prefers $x = (x_1, x_2) \in X$ to $y = (y_1, y_2) \in X$ iff $x_i > y_i$

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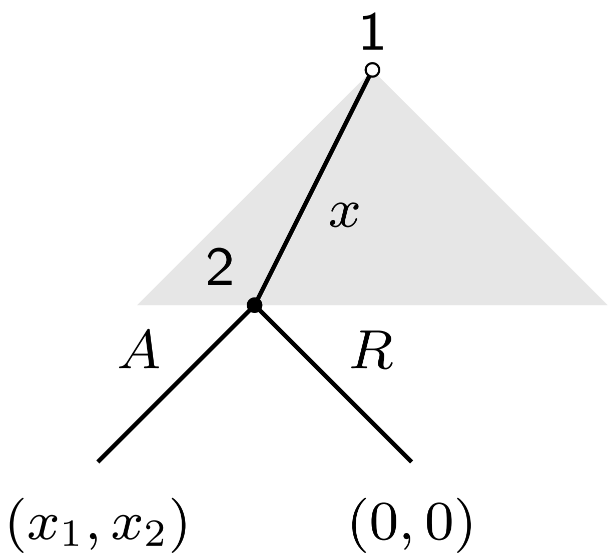

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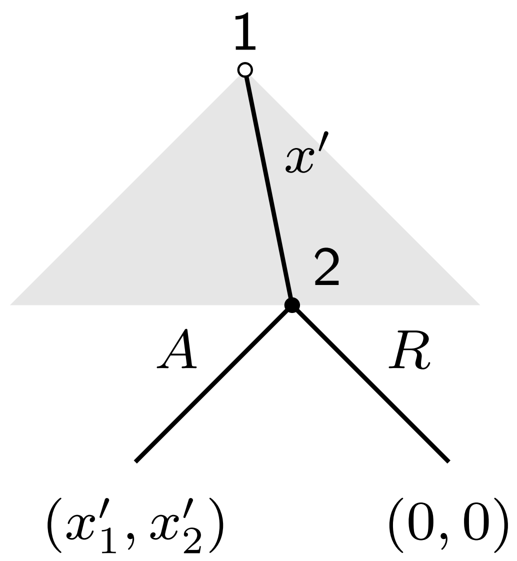

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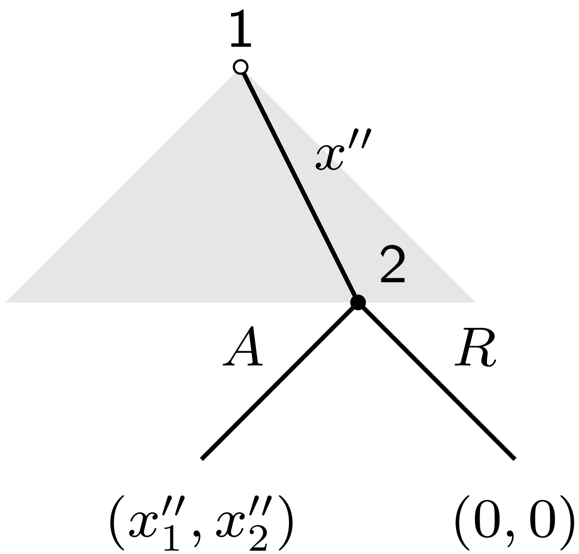

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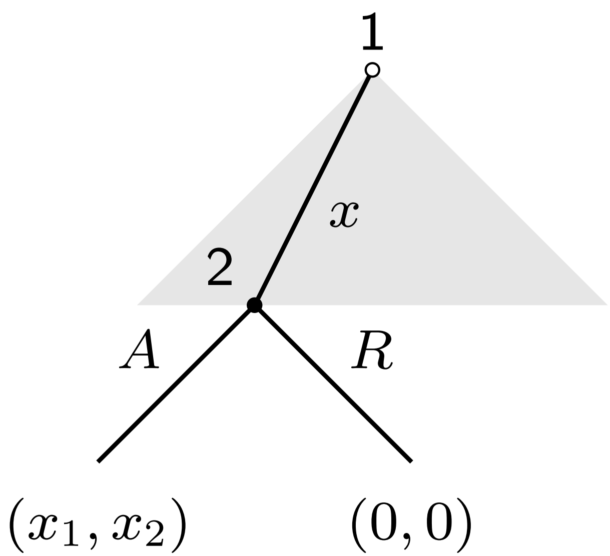

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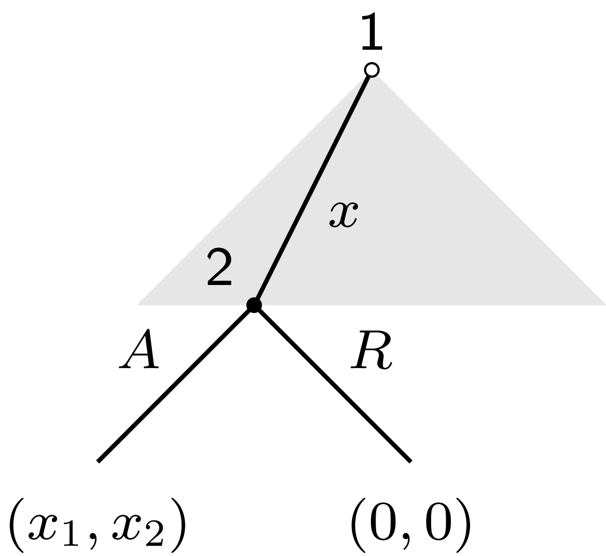

✍ Is every agreement a Nash equilibrium outcome?

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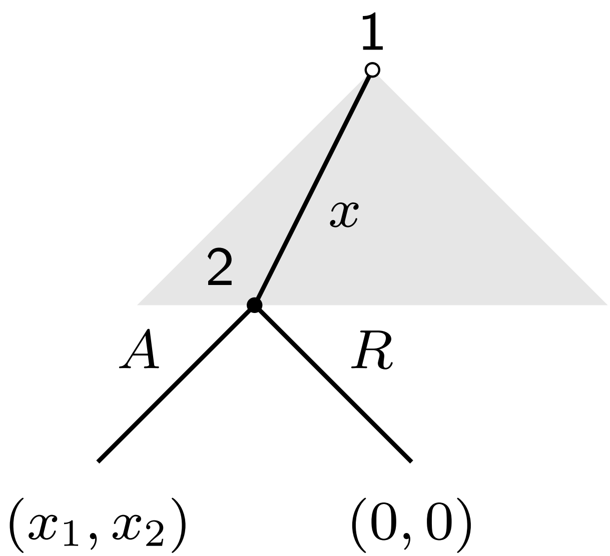

✍ Is every agreement a Nash equilibrium outcome?

Unique SPNE: player 1 proposes $(1, 0)$ and player 2 accepts every offer

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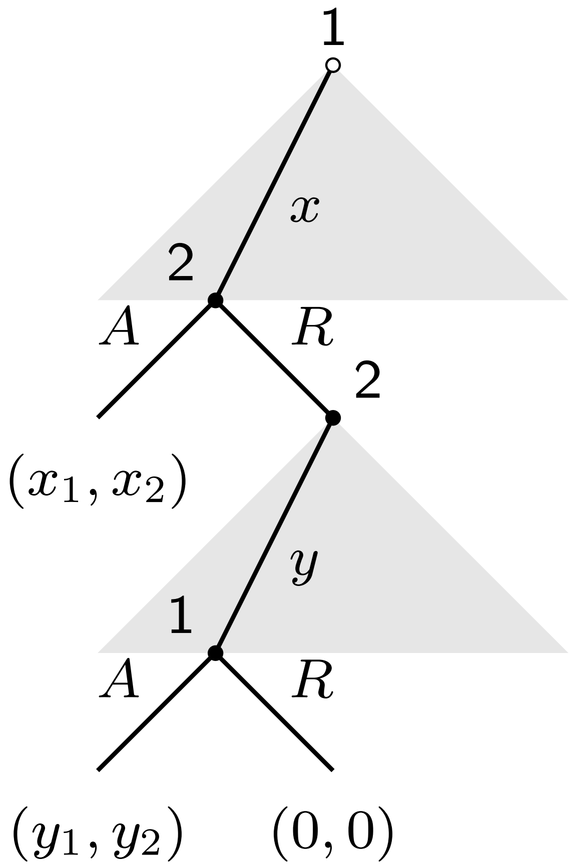

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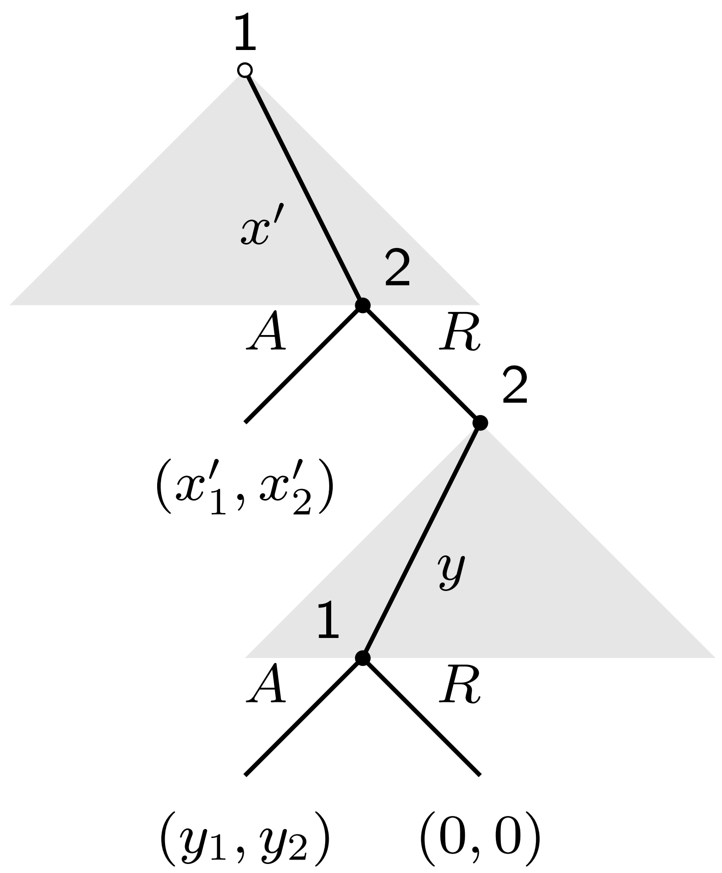

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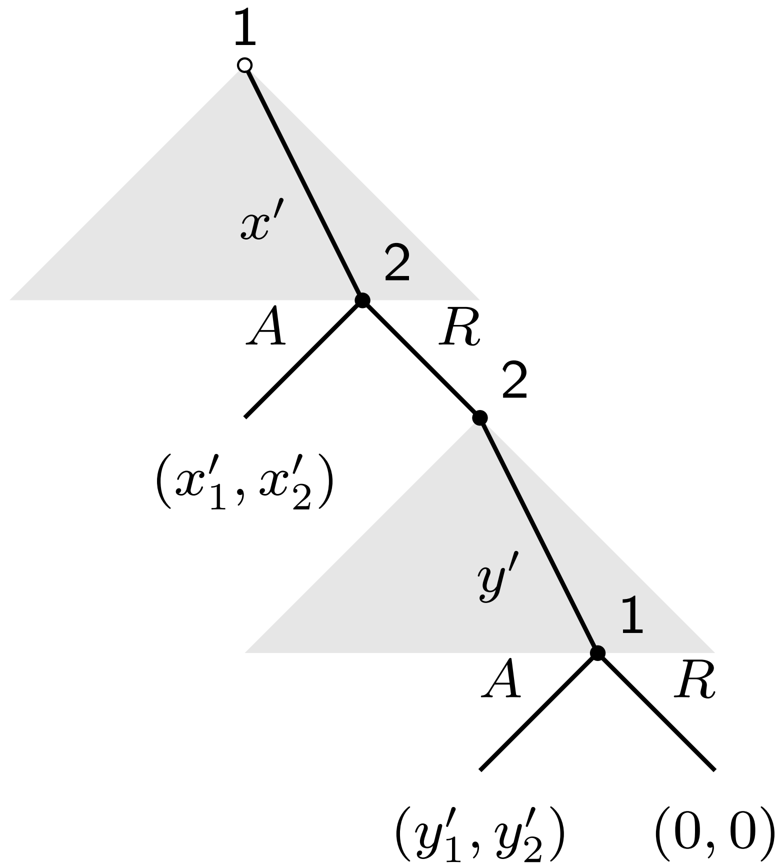

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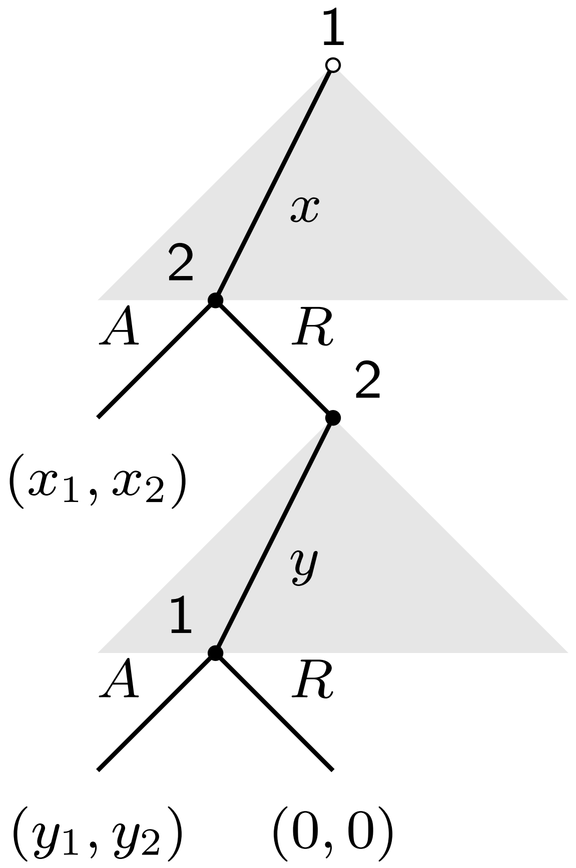

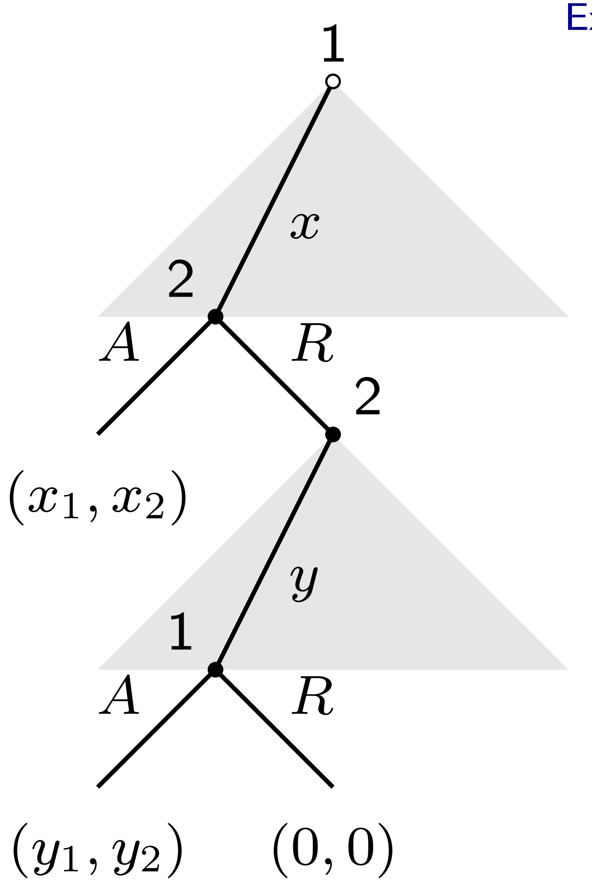
1
x
2
A
R
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y
1
A
R
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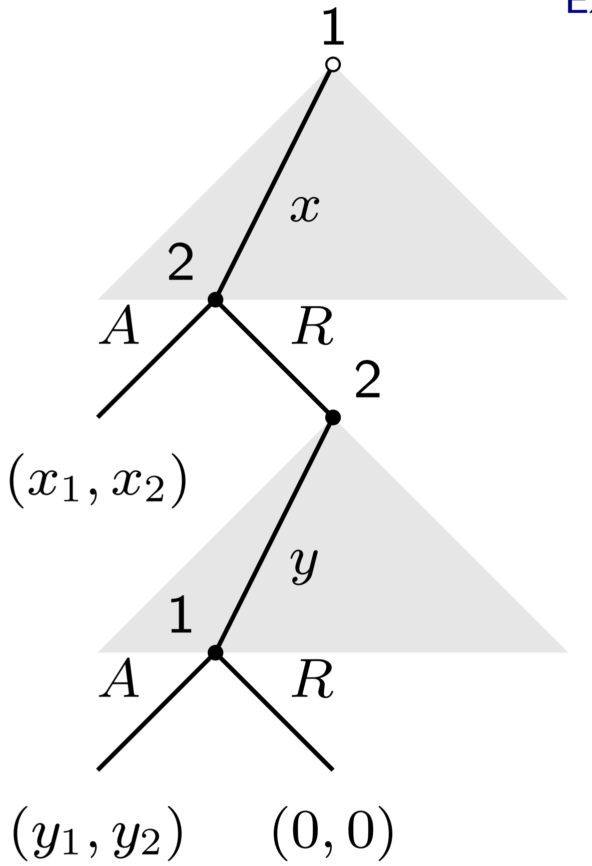
$(0, 0)$



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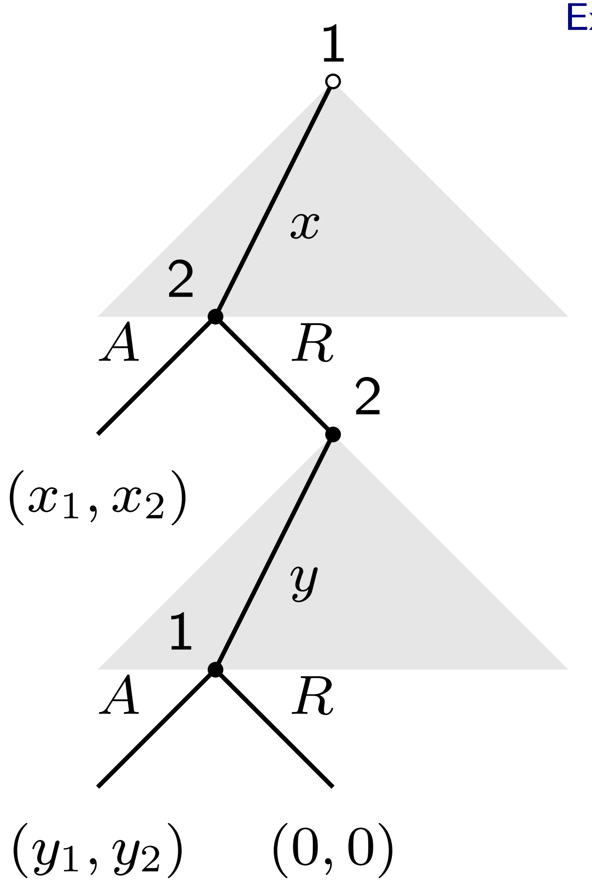

Now, it is player 2 who has all the bargaining power

Backward induction $\Rightarrow$ solution $y = (0, 1)$ and A in the second period

$\Rightarrow$ $x = (0, 1)$, or $x \neq (0, 1)$ and R in the first period

$\Rightarrow$ at every SPNE player 2 obtains all the pie

More generally, whatever the length of the game, the player who makes the last offer obtains all the pie

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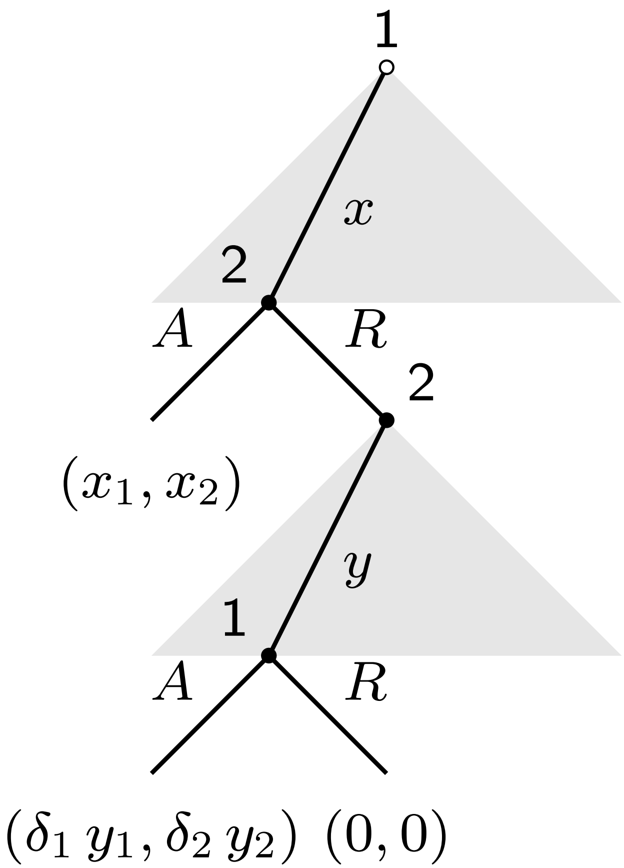

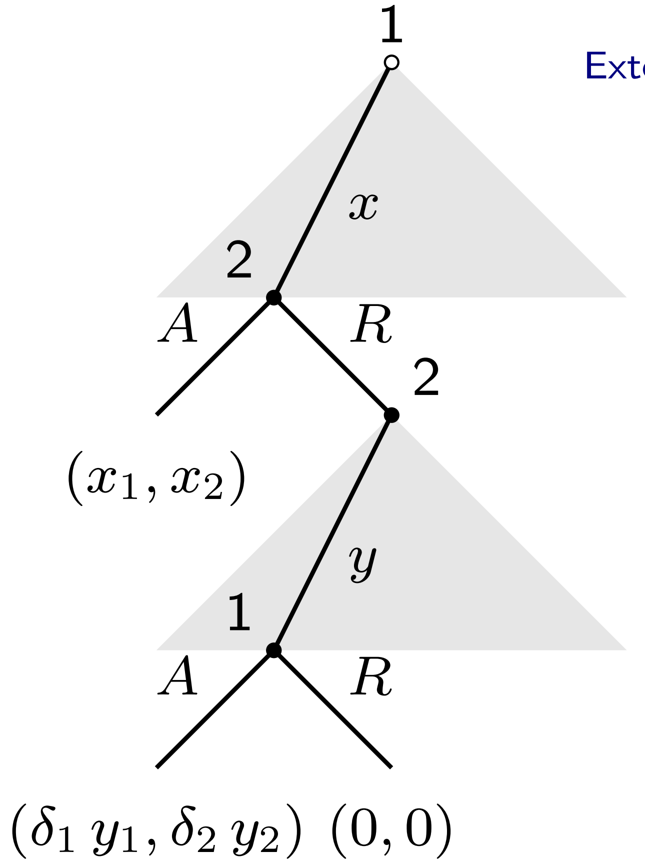

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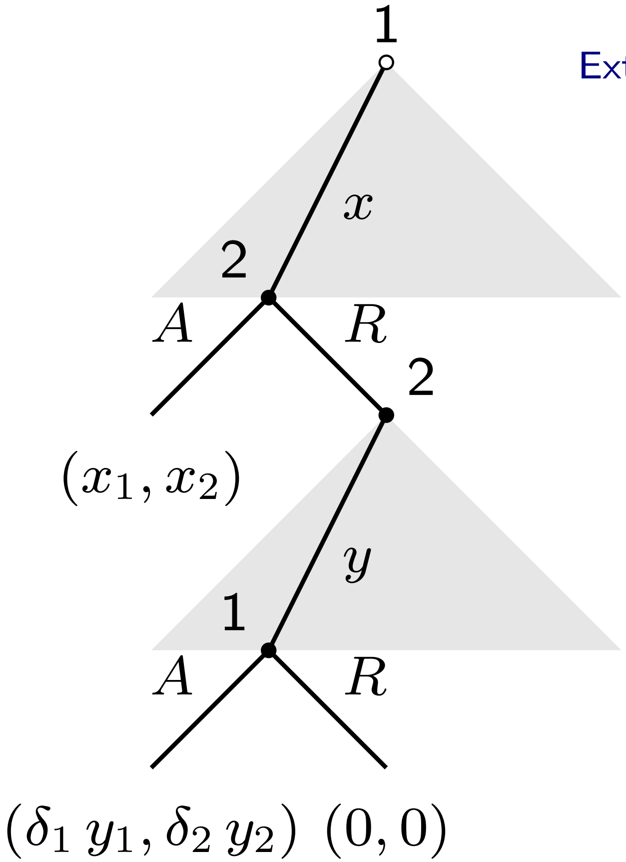
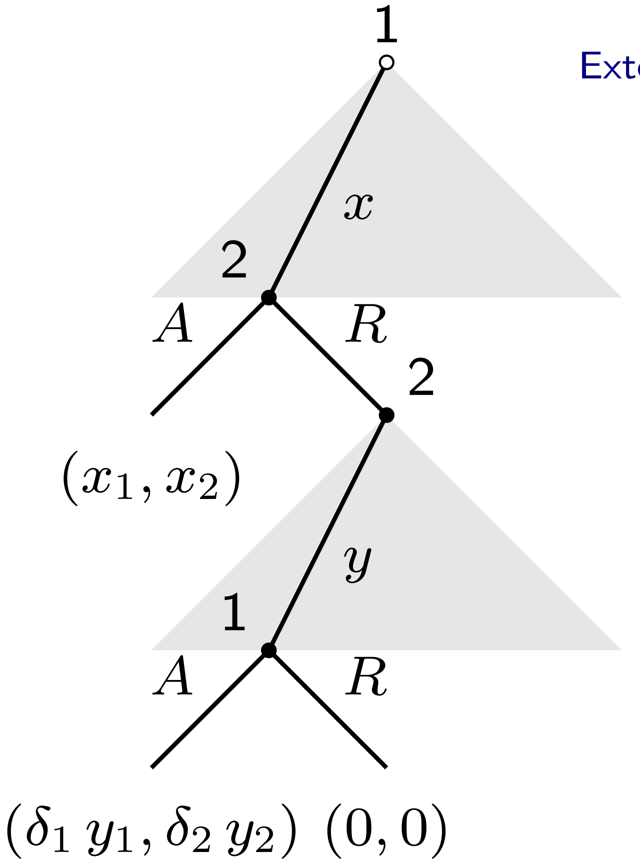


Backward induction:

Subgame after player 2's rejection: unique SPNE: player 2 proposes $(0, 1)$ and player 1 accepts every offer $\Rightarrow$ payoff $(0, \delta_2)$



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Subgame after player 2's rejection: unique SPNE: player 2 proposes $(0, 1)$ and player 1 accepts every offer $\Rightarrow$ payoff $(0, \delta_2)$

Subgame after player 1's proposal: player 2 accepts $x_2 \geq \delta_2$ and rejects $x_2 < \delta_2 \Rightarrow$ player 1 proposes $(x_1, x_2) = (1 - \delta_2, \delta_2)$ in the first period

Finite Horizon Bargaining

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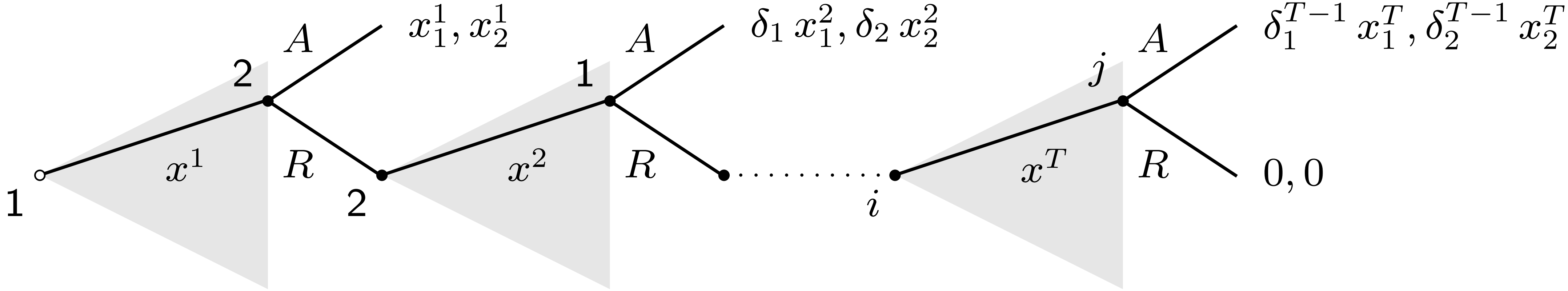

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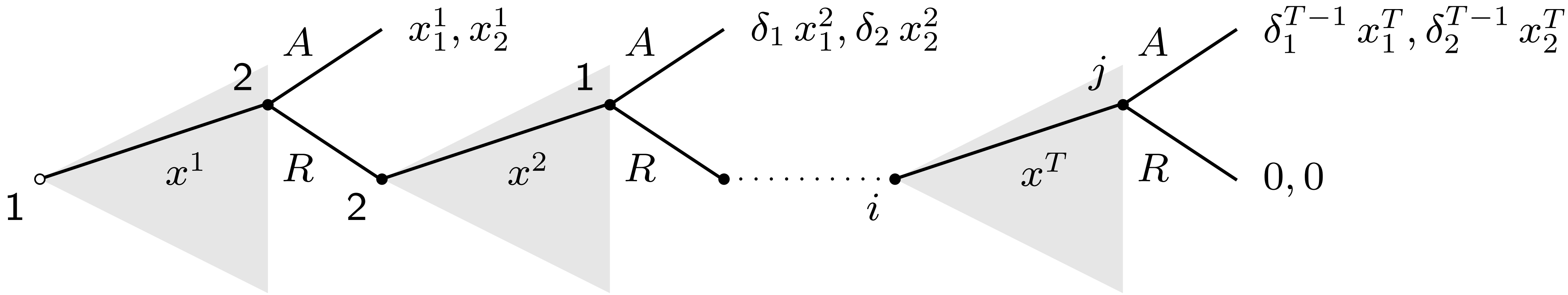

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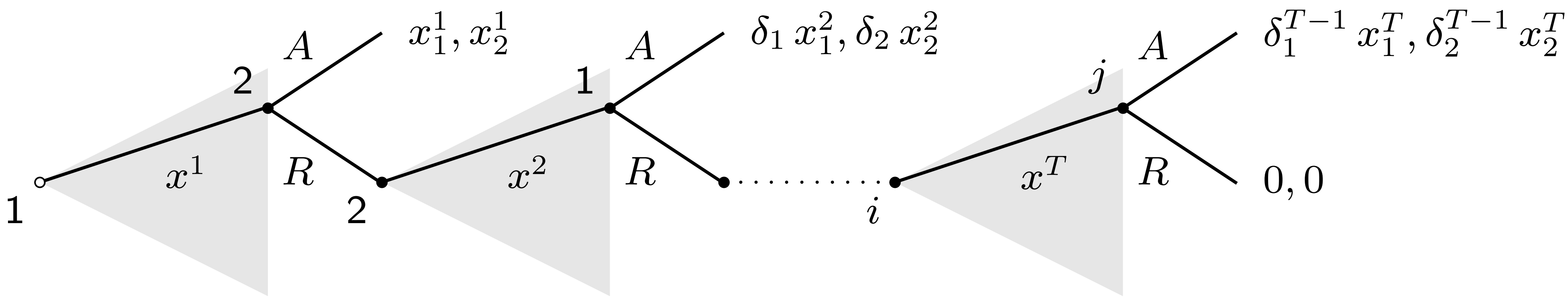

$i\ (j) =$ player 1 if T is odd (even) $\qquad i\ (j) =$ player 2 if T is even (odd)

➥ Backward induction

✍ Check that if $T = 3$ then $x^1 = (1 - \delta_2(1 - \delta_1), \delta_2(1 - \delta_1))$

✍ Check that if $T = 4$ then $x^1 = (1 - \delta_2(1 - \delta_1(1 - \delta_2)), \delta_2(1 - \delta_1(1 - \delta_2)))$

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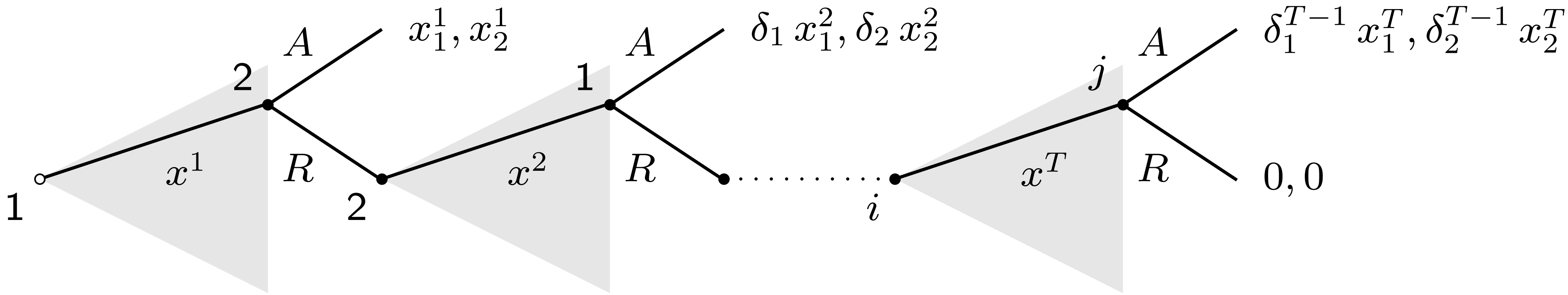

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Problem: the solution depends significantly on the exact deadline

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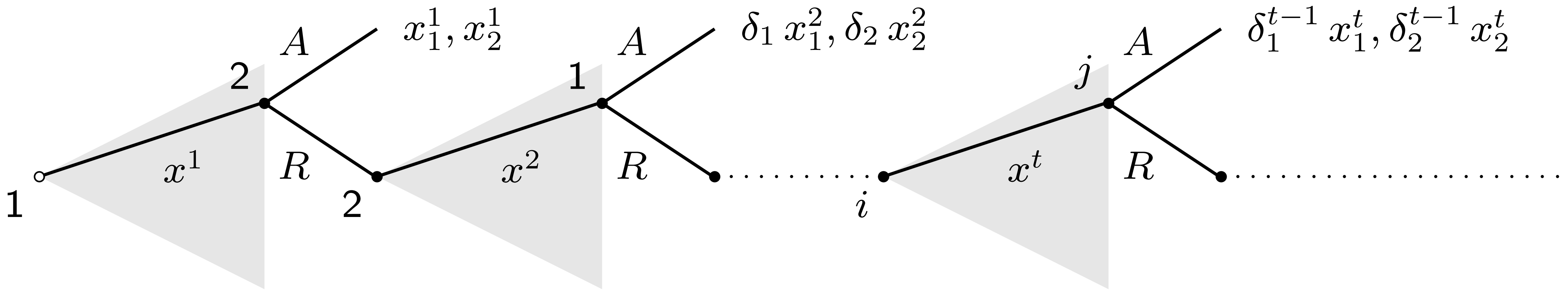

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- Unique asymmetry in the game tree: player 1 is the first to make an offer
- It is common knowledge that players only care about the final agreement x and the period at which this agreement is reached (very strong assumption)
- The structure of the game is repeated, but it is not a repeated game ($A \Rightarrow$ end of the "repetition")

Pure strategy of player 1: Sequence $\sigma = (\sigma^t)_{t=1}^{\infty}$, where

$$\sigma^t : X^{t-1} \to X \quad \text{if } t \text{ is odd}$$

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Stationary strategies: do not depend on the period and on past offers

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$$\sigma^t(x^{t-1}) = \begin{cases} A & \text{if } x_1^{t-1} \geq \overline{x}_1 \\ R & \text{if } x_1^{t-1} < \overline{x}_1 \end{cases} \qquad \text{if } t \text{ is even}$$

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Accepted offers at the SPNE: $\forall\ t$, $\forall\ \delta < 1$ ➠ $y_1^* = \overline{x}_1$ and $x_2^* = \overline{y}_2$

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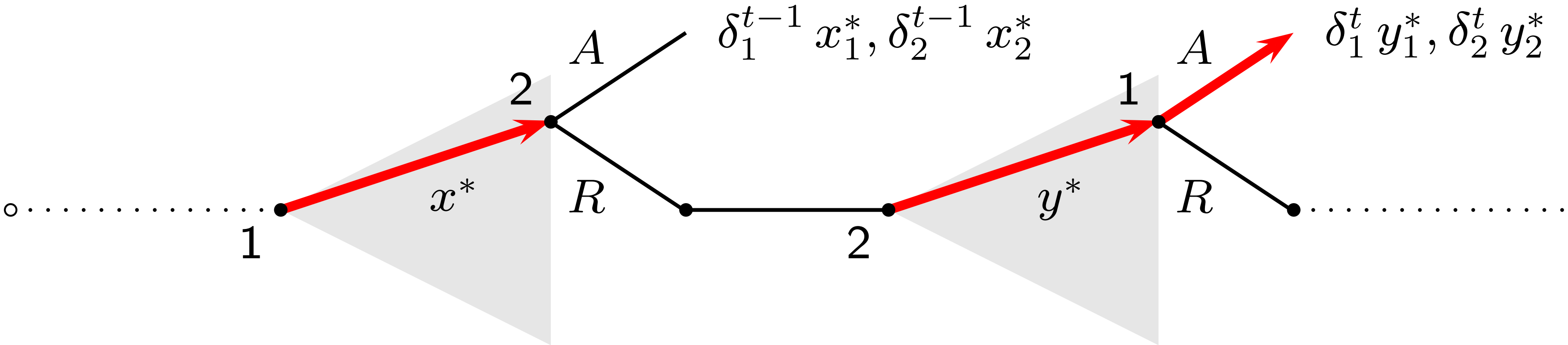

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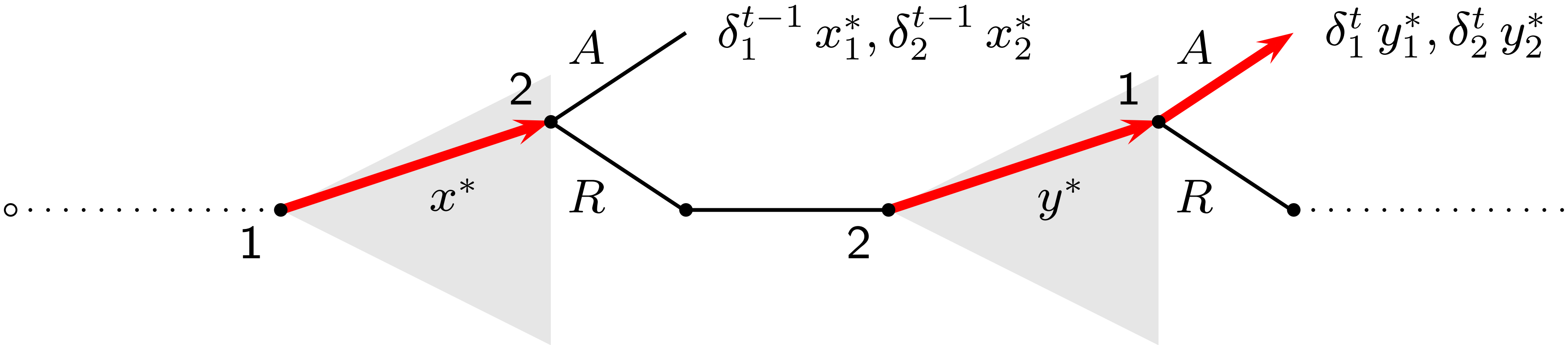

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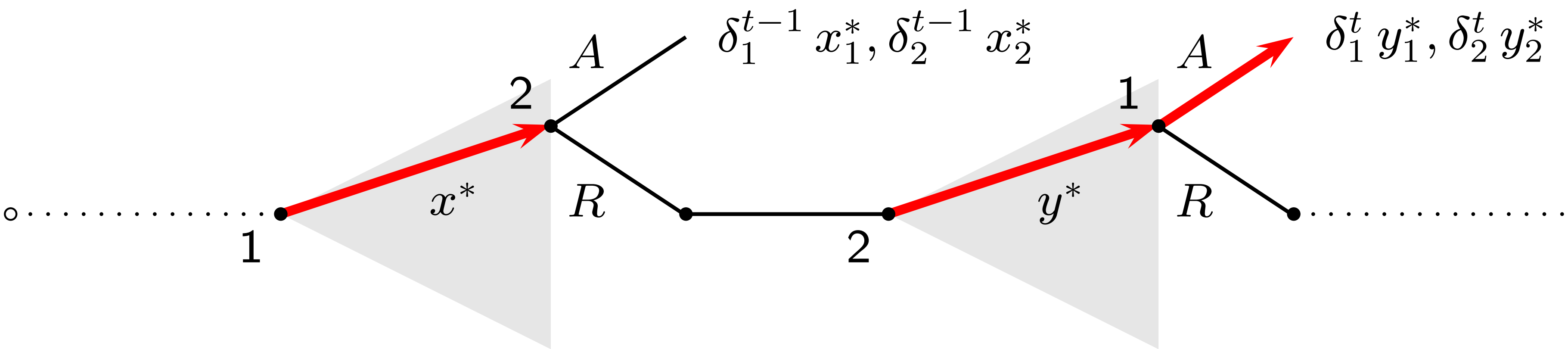

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Symmetric reasoning for player 1 $\Rightarrow$ $y_1^* = \delta_1\,x_1^*$

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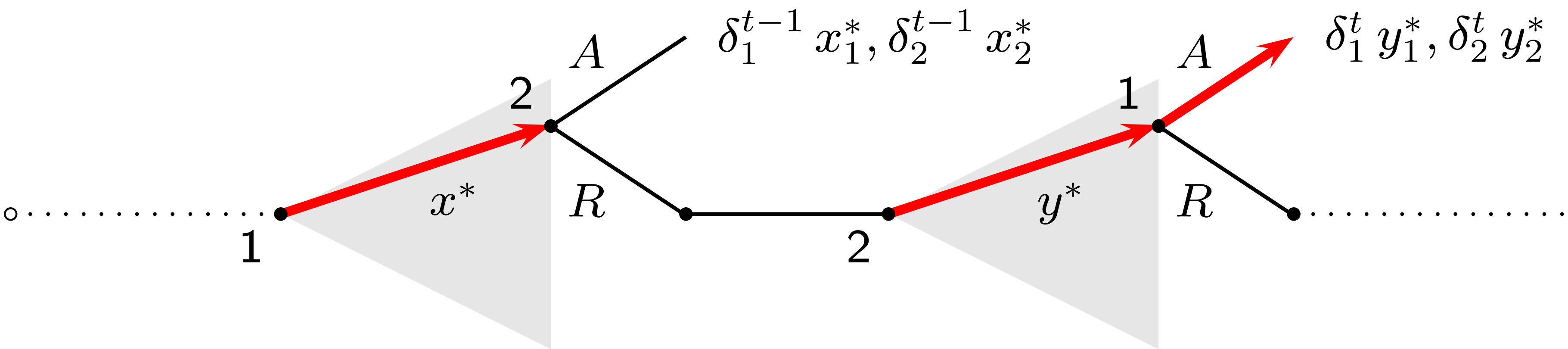

Equilibrium $\Rightarrow$ $\delta_2^{t-1}\,x_2^* = \delta_2^t\,y_2^*$, i.e., $x_2^* = \delta_2\,y_2^*$

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➢ If only one player is able to make offers then, at a SPNE, he obtains all the pie in the first period

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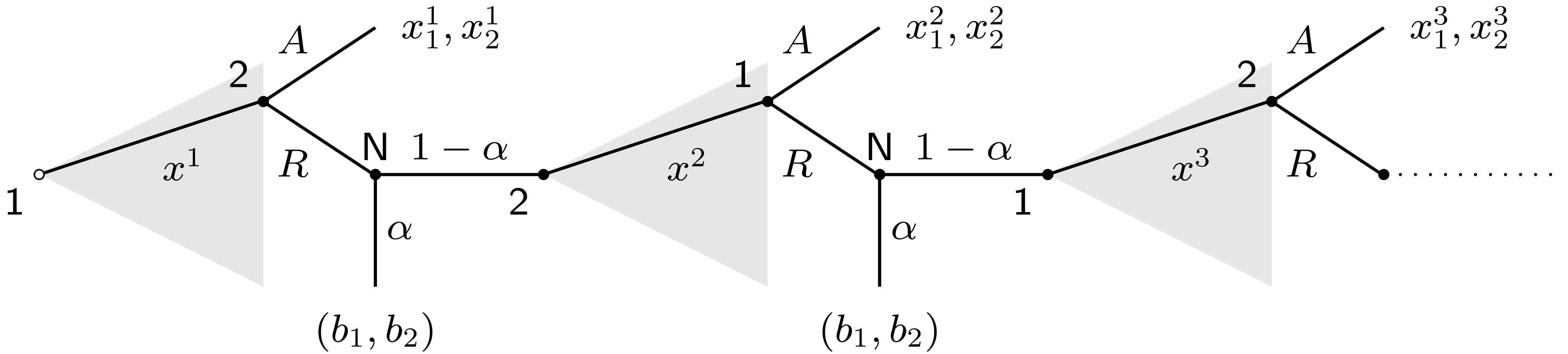

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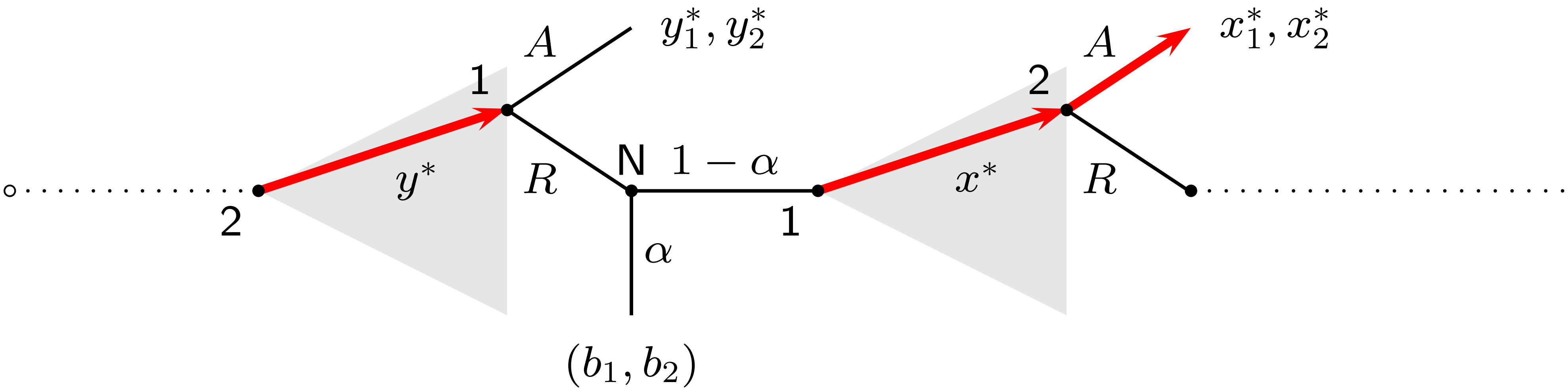

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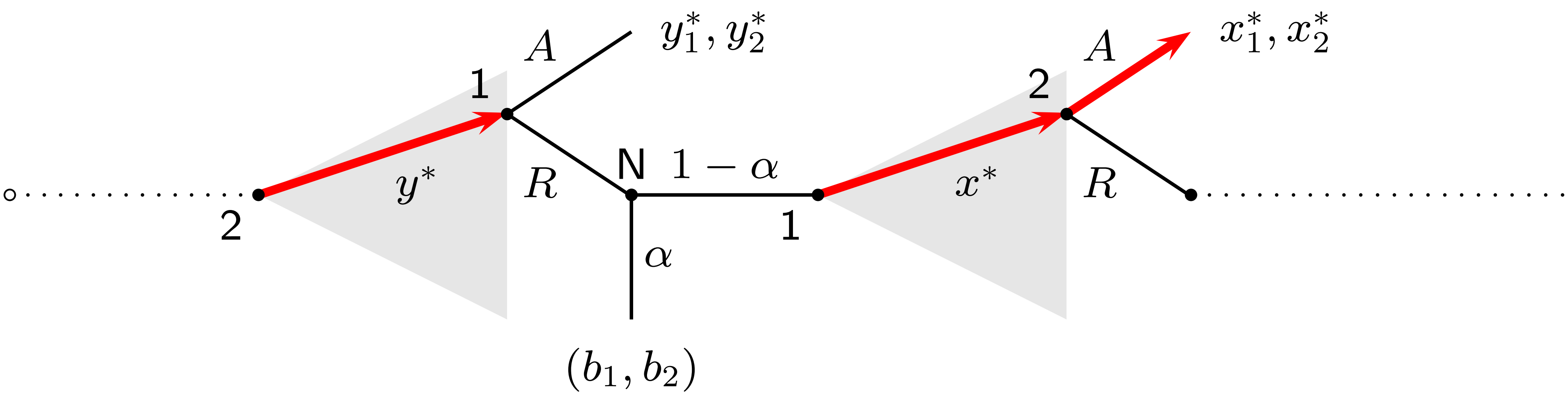

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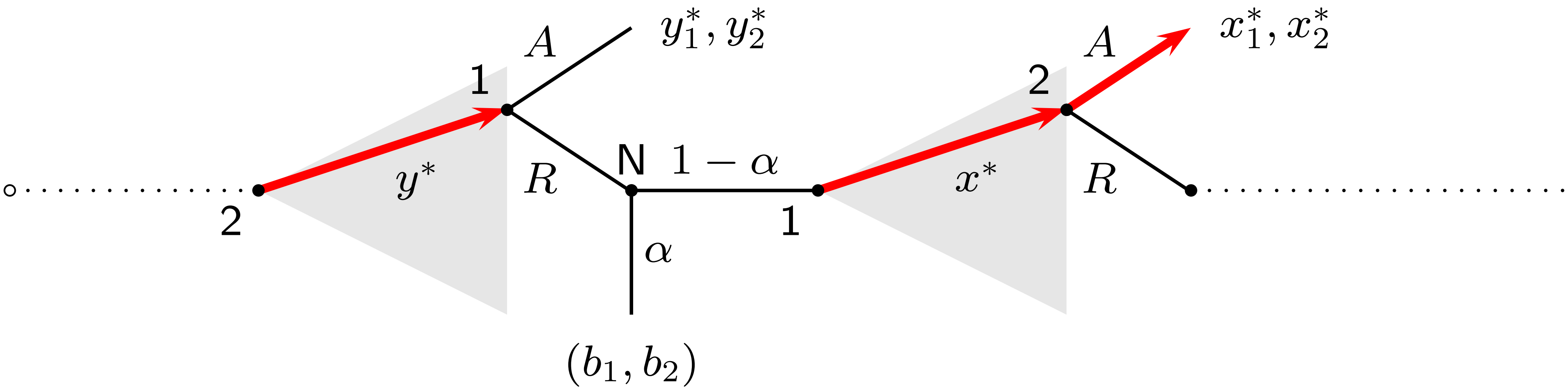

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➥ Each player gets his payoff in the event of breakdown (b_i) and we split equally the excess of the pie $(\frac{1-b_1-b_2}{2})$

References

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