Exercise 5 **[1] Each day $t = 1, 2, 3 \ldots$, two traders can buy (action B) or sell (action S) some assets of a company (prices and quantities are not explicitly characterized). There are nine possible states of the world:

$$\Omega = \{\omega_1, \omega_2, \ldots, \omega_9\}.$$

Each state has the same prior probability ($\Pr(\omega) = 1/9 \ \forall \ \omega \in \Omega$), common to both traders. Traders' initial information partitions are given by:

$$\mathcal{P}_1 = \{\{\omega_1, \omega_2, \omega_3\}, \{\omega_4, \omega_5, \omega_6\}, \{\omega_7, \omega_8, \omega_9\}\}$$
$$\text{and } \mathcal{P}_2 = \{\{\omega_1, \omega_2, \omega_3, \omega_4\}, \{\omega_5, \omega_6, \omega_7, \omega_8\}, \{\omega_9\}\}.$$

Let $E = \{\omega_1, \omega_5, \omega_9\}$ be the set of states of the world in which the company earnings will go down (bad outcome). Suppose that each trader behaves each day according to the following rule:

- *Buy* if he believes with probability strictly less than 0.3 that E is true
- *Sell* if he believes with probability more than 0.3 that E is true.

Denote by $\mathcal{P}_i^t$ the information partition of trader i ($i = 1, 2$) at the beginning of period t ($t = 1, 2, 3, \ldots$). Hence we have:

$$\mathcal{P}_1^1 = \mathcal{P}_1 = \{\{\omega_1, \omega_2, \omega_3\}, \{\omega_4, \omega_5, \omega_6\}, \{\omega_7, \omega_8, \omega_9\}\}$$
$$\text{and } \mathcal{P}_2^1 = \mathcal{P}_2 = \{\{\omega_1, \omega_2, \omega_3, \omega_4\}, \{\omega_5, \omega_6, \omega_7, \omega_8\}, \{\omega_9\}\}.$$

(1) At the beginning of the first period, is there a state in which one of the traders knows that the company has bad outcomes? Is there a state in which it is commonly known that the company has bad outcomes?

(2) Determine traders' beliefs about E at each of their information sets before any transaction takes place ($t = 1$). Deduce traders' optimal actions (buy or sell) in each state.

(3) At the beginning of the first period, explain why traders' information partitions become

$$\mathcal{P}_1^2 = \{\{\omega_1, \omega_2, \omega_3\}, \{\omega_4, \omega_5, \omega_6\}, \{\omega_7, \omega_8\}, \{\omega_9\}\}$$
$$\text{and } \mathcal{P}_2^2 = \{\{\omega_1, \omega_2, \omega_3, \omega_4\}, \{\omega_5, \omega_6, \omega_7, \omega_8\}, \{\omega_9\}\}.$$

(4) At the beginning of the second period, find the set of states in which it is commonly known that the state is not ω_9. Show that in the first period, if the real state was ω_1, it was mutually known at order 3 that the state was not ω_9 but it was not commonly known.

(5) Determine traders' beliefs about E at each of their information sets at the beginning of period $t = 2$. Deduce traders' optimal actions (buy or sell) in each state.

(6) Characterize traders' information partitions $\mathcal{P}_1^3$ et $\mathcal{P}_2^3$ at the beginning of period $t = 3$ (after having observed the transactions in period 2). Deduce traders' optimal actions (buy or sell) in each state of period 3.

(7) Do the same for the two next periods ($t = 4$ and $t = 5$). Do partitions evolve after period 5? Why?

(8) Deduce from the previous questions the dynamic of traders' actions when the real state is ω_1. Same question when the real state is ω_4. Comment.

[1] Adapted from Hart and Tauman (2004) "Market Crashes without Exogenous Shocks", *The Journal of Business*, 2004.