Strategic Information Transmission: Cheap Talk Games

Outline
(November 12, 2008)

1/

- Credible information under cheap talk: Examples
- Geometric characterization of Nash equilibrium outcomes
- Expertise with a biased interested party
- Communication in organizations: Delegation vs. cheap talk vs. commitment
- Multiple Senders and Multidimensional Cheap Talk
- Lobbying with several audiences
- Some experimental evidence

General References:

- Bolton and Dewatripont (2005, chap. 5) "Disclosure of Private Certifiable Information," in "Contract Theory"
- Farrell and Rabin (1996): "Cheap Talk," Journal of Economic Perspectives
- Forges (1994): "Non-Zero Sum Repeated Games and Information Transmission," in Essays in Game Theory: In Honor of Michael Maschler

2/

- Koessler and Forges (2006): "Multistage Communication with and without Verifiable Types", International Journal of Game Theory
- Kreps and Sobel (1994) : "Signalling," in "Handbook of Game Theory" vol. 2
- Myerson (1991, chap. 6): "Games of communication," in "Game Theory, Analysis of Conflict"
- Sobel (2007): "Signalling Games"

Cheap talk = communication which is

- strategic and non-binding (no contract, no commitment)
- costless, without *direct* impact on payoffs
- direct / face-to-face / unmediated
- possibly several communication stages
- soft information (not verifiable, not certifiable, not provable)

3/

⇒ different, e.g., from information revelation by a price system in rational expectation general equilibrium models (Radner, 1979), from mechanism design (contract), from signaling à la Spence (1973),...

In its simplest form, a cheap talk game in a specific signaling games in which messages are costless (i.e., do not enter into players' utility functions)

Example 1. (Signal of productivity in a labor market)

Extremely simplified version of Spence (1973) model of education:

The sender (the expert) is a worker with private information about his ability $k \in \{k_L, k_H\} = \{1, 3\}$

The receiver (the decisionmaker) is an employer who must chose a salary $j \in \{j_L, j_M, j_H\} = \{1, 2, 3\}$

The worker's productivity is assumed to be equal to his ability

4/

Perfect competition among employers, so the employer chooses a salary equal to the expected productivity of the worker (zero expected profits)

The worker chooses a level of education $e \in \{e_L, e_H\} = \{0, 3\}$ (which does no affect his productivity, but is *costly*)

$$\begin{cases} A^k(j) = j - c(k,e) = j - e/k & \text{(worker)} \\ B^k(j) = -\big[k - j\big]^2 & \text{(employer)} \end{cases}$$

5/

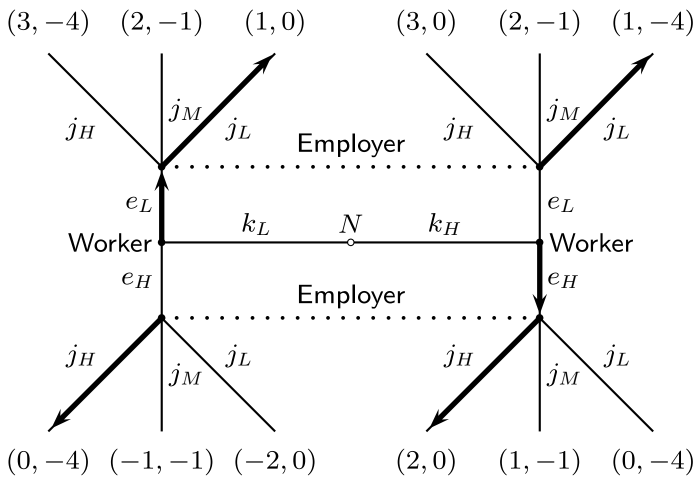


Figure 1: Fully revealing equilibrium in the labor market signaling game (example 1)

What happens if we replace the level of education e by cheap talk?

Then, the message "my ability is high" is not credible anymore: whatever his type, the worker always wants the employer to believe that his ability is high (in order to get a high salary)

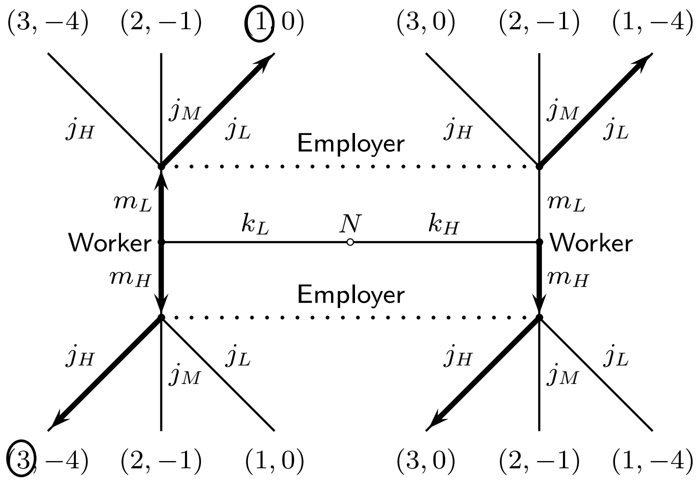
6/

	$j_H = 3$	$j_M = 2$	$j_L = 1$	
k_L	$3, -4$	$2, -1$	$1, 0$	$\Pr(k_L) = 1/2$

	$j_H = 3$	$j_M = 2$	$j_L = 1$	
k_H	$3, 0$	$2, -1$	$1, -4$	$\Pr(k_H) = 1/2$

Associated one-shot cheap talk game with two possible messages

7/



Fully revealing equilibrium? No, because the worker of type k_L deviates by sending the same message as the worker of type k_H

✍ Non-revealing equilibrium? Yes, a NRE always exists in cheap talk games

Can cheap talk be credible and help to transmit relevant information?

Example 2. (Credible information revelation)

8/

	j_1	j_2	
k_1	$1,1$	$0,0$	p
k_2	$0,0$	$3,3$	$(1-p)$

$$Y(p) = \begin{cases} \{j_1\} & \text{if } p > 3/4, \\ \{j_2\} & \text{if } p < 3/4, \\ \Delta(J) & \text{if } p = 3/4. \end{cases}$$

The sender's preference over the receiver's beliefs are **positively correlated** with the truth

9/

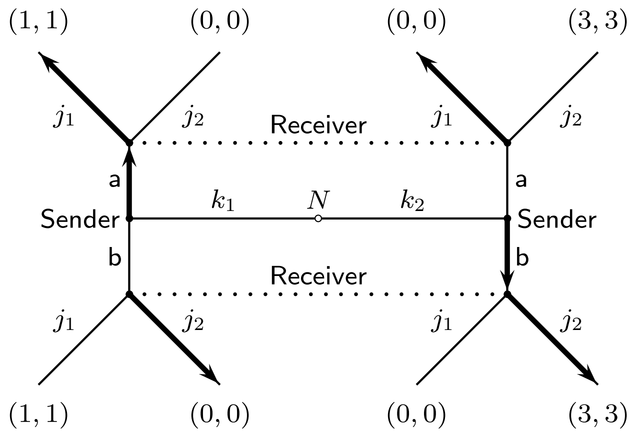


Figure 2: Fully revealing equilibrium in Example 2.

Example 3. (Revelation of information which is not credible)

	j_1	j_2	
k_1	$5, 2$	$1, 0$	p
k_2	$3, 0$	$1, 4$	$(1-p)$

10/

$$Y(p) = \begin{cases} \{j_1\} & \text{if } p > 2/3, \\ \{j_2\} & \text{if } p < 2/3, \\ \Delta(J) & \text{if } p = 2/3. \end{cases}$$

The sender's preference over the receiver's beliefs is **not correlated** with the truth.
The unique equilibrium of the cheap talk game in NR, even if when $p < 2/3$ communication of information would increase both players' payoffs

Example 4. (Revelation of information which is not credible)

	j_1	j_2	
k_1	$3, 2$	$4, 0$	p
k_2	$3, 0$	$1, 4$	$(1-p)$

11/

$$Y(p) = \begin{cases} \{j_1\} & \text{if } p > 2/3, \\ \{j_2\} & \text{if } p < 2/3, \\ \Delta(J) & \text{if } p = 2/3. \end{cases}$$

The sender's preference over the receiver's beliefs is **negatively correlated** with the truth. The unique equilibrium of the cheap talk game in NR

Example 5. (Partial revelation of information)

	j_1	j_2	j_3	j_4	j_5	
k_1	$1, 10$	$3, 8$	$0, 5$	$3, 0$	$1, -8$	p
k_2	$1, -8$	$3, 0$	$0, 5$	$3, 8$	$1, 10$	$1-p$

12/

$$Y(p) = \begin{cases} \{j_5\} & \text{if } p < 1/5 \\ \{j_4\} & \text{if } p \in \ (1/5, 3/8) \\ \{j_3\} & \text{if } p \in \ (3/8, 5/8) \\ \{j_2\} & \text{if } p \in \ (5/8, 4/5) \\ \{j_1\} & \text{if } p > 4/5 \end{cases}$$

Partially revealing equilibrium when $p = 1/2$:

$$\begin{cases} \sigma(k_1) = \frac{3}{4}\mathsf{a} + \frac{1}{4}\mathsf{b} \\ \sigma(k_2) = \frac{1}{4}\mathsf{a} + \frac{3}{4}\mathsf{b} \end{cases}$$

13/

$$\Rightarrow \begin{cases} \Pr(k_1 \mid \mathsf{a}) = \frac{\Pr(\mathsf{a} \mid k_1)\Pr(k_1)}{\Pr(\mathsf{a})} = 3/4 \\ \Pr(k_1 \mid \mathsf{b}) = \frac{\Pr(\mathsf{b} \mid k_1)\Pr(k_1)}{\Pr(\mathsf{b})} = 1/4 \end{cases}$$

$$\Rightarrow \begin{cases} \tau(\mathsf{a}) = j_2 \\ \tau(\mathsf{b}) = j_4 \end{cases}$$

$\Rightarrow$ equilibrium, expected utility $= \frac{3}{4}(3,8) + \frac{1}{4}(3,0) = (3,6)$ (better for the sender than the NRE and FRE)

Basic Decision Problem

Two players

Player 1 = sender, expert (with no decision)

Player 2 = receiver, decisionmaker (with no information)

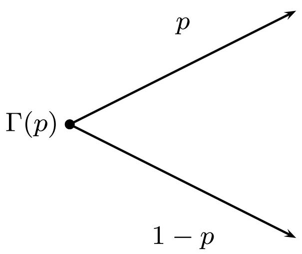
14/

Two possible types for the expert (can be easily generalized): $K = \{k_1, k_2\} = \{1, 2\}$, $\Pr(k_1) = p$, $\Pr(k_2) = 1 - p$

Action of the decisionmaker: $j \in J$

Payoffs: $A^k(j)$ and $B^k(j)$

Silent Game



	1	$\cdots$	j	$\cdots$
k_1	$A^1(1),\, B^1(1)$	$\cdots$	$A^1(j),\, B^1(j)$	$\cdots$

15/

	1	$\cdots$	j	$\cdots$
k_2	$A^2(1),\, B^2(1)$	$\cdots$	$A^2(j),\, B^2(j)$	$\cdots$

- Mixed action of the DM: $y \in \Delta(J)$

$$\Rightarrow \text{ expected payoffs } \begin{cases} A^k(y) = \sum_{j \in J} y(j)\, A^k(j) \\ B^k(y) = \sum_{j \in J} y(j)\, B^k(j) \end{cases}$$

- Optimal mixed actions in $\Gamma(p)$ (non-revealing "equilibria"):

16/

$$\begin{aligned} Y(p) &\equiv \arg\max_{y \in \Delta(J)} p\, B^1(y) + (1-p)\, B^2(y) \\ &= \{y : p\, B^1(y) + (1-p)\, B^2(y) \geq p\, B^1(j) + (1-p)\, B^2(j),\ \forall\, j \in J\} \end{aligned}$$

Remark Mixed actions are used in the communication extension of the game to construct equilibria in which the expert is indifferent between several messages. They also serve as punishments off the equilibrium path in communication games with certifiable information (persuasion games)

- "Equilibrium" payoffs in $\Gamma(p)$:

$$\mathcal{E}(p) \equiv \{(a, \beta) : \exists\, y \in Y(p),\ a = A(y),\ \beta = p\, B^1(y) + (1-p)\, B^2(y)\}$$

17/

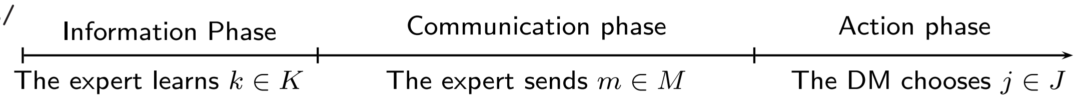
Unilateral Communication Game $\Gamma_S^0(p)$

Unilateral information transmission from the expert to the decisionmaker

Set of *messages* ("keyboard") of the expert:

$$M = \{a, b, \ldots, \}, \qquad 3 \leq |M| < \infty$$

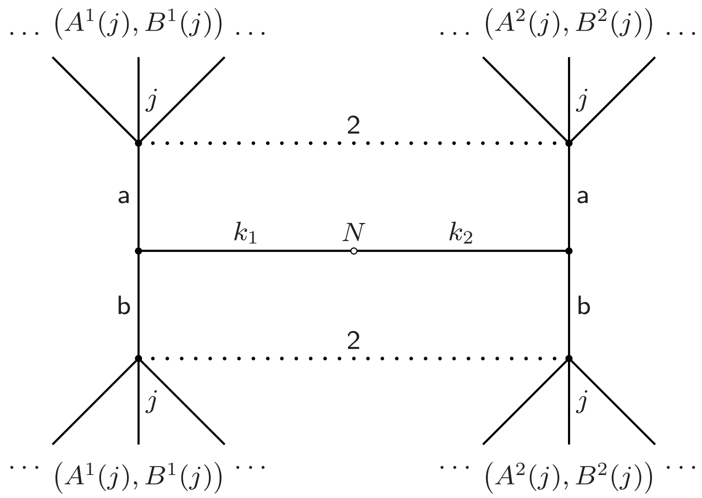
18/



Strategy of the expert: $\sigma : K \to \Delta(M)$

Strategy of the DM: $\tau : M \to \Delta(J)$

Example. Two messages ($M = \{a, b\}$)



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$\mathcal{E}^0_S(p)$: Equilibrium payoffs of $\Gamma^0_S(p)$

Characterization of NE Payoffs of $\Gamma^0_S(p)$

Recall. $\mathcal{E}(p) \subseteq \mathbb{R}^2 \times \mathbb{R}$: NE payoffs in the silent game $\Gamma(p)$

Modified equilibrium payoffs of $\Gamma(p)$: $\mathcal{E}^+(p)$: the expert can have a (virtual) payoff which is higher than his equilibrium payoff when his type has zero probability

➥ $(a, \beta) \in \mathbb{R}^2 \times \mathbb{R}$ such that there exists an optimal action $y \in Y(p)$ in the silent game $\Gamma(p)$ satisfying

20/ (i) $a^k \geq A^k(y)$, for all $k \in K$

(ii) $a^1 = A^1(y)$ if $p \neq 0$ and $a^2 = A^2(y)$ if $p \neq 1$

(iii) $\beta = p\, B^1(y) + (1-p)\, B^2(y)$

(Thus, $\mathcal{E}^+(p) = \mathcal{E}(p)$ if $p \in (0,1)$)

Graph of the modified equilibrium payoff correspondence:

$$\mathrm{gr}\, \mathcal{E}^+ \equiv \{(a, \beta, p) \in \mathbb{R}^2 \times \mathbb{R} \times [0,1] : (a, \beta) \in \mathcal{E}^+(p)\}$$

Hart (1985, MOR), Aumann and Hart (2003, Ecta): Without any assumption on the utility functions, all equilibrium payoffs of the unilateral communication game $\Gamma_S^0(p)$ can be geometrically characterized only from the graph of the equilibrium payoff correspondence of the silent game

21/ **Theorem (Characterization of $\mathcal{E}_S^0(p)$)** *Let $p \in (0,1)$. A payoff profile (a, β) is a Nash equilibrium payoff of the unilateral communication game $\Gamma_S^0(p)$ if and only if (a, β, p) belongs to* $\text{conv}_a(\text{gr}\,\mathcal{E}^+)$*, the set points obtained by convexification of the set $\text{gr}\,\mathcal{E}^+$ in (β, p) by keeping the expert's payoff, a, constant:*

$$\mathcal{E}_S^0(p) = \{(a, \beta) \in \mathbb{R}^2 \times \mathbb{R} : (a, \beta, p) \in \text{conv}_a(\text{gr}\,\mathcal{E}^+)\}$$

Illustrations

Unique equilibrium, non revealing (Example 1)

Optimal decisions in the silent game:

22/

$$Y(p) = \begin{cases} \{j_H\} & \text{if } p < 1/4 \\ \Delta(\{j_H, j_M\}) & \text{if } p = 1/4 \\ \{j_M\} & \text{if } p \in (1/4, 3/4) \\ \Delta(\{j_M, j_L\}) & \text{if } p = 3/4 \\ \{j_L\} & \text{if } p > 3/4 \end{cases}$$

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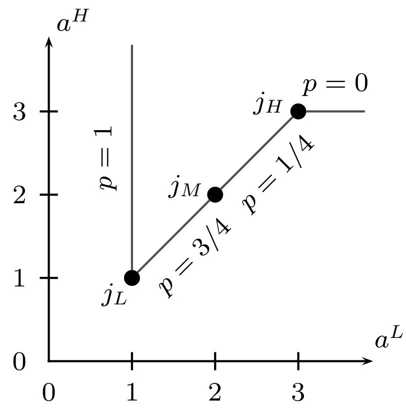


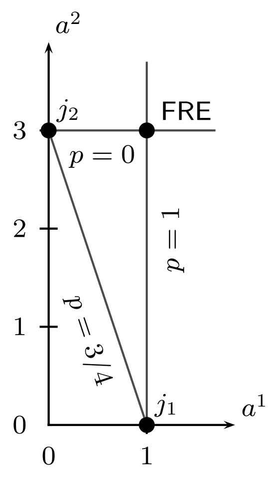
Figure 3: Modified equilibrium payoffs in Example 1

Full revelation of information (Example 2)

24/

	j_1	j_2	
k_1	$1,1$	$0,0$	p
k_2	$0,0$	$3,3$	$(1-p)$

$$Y(p) = \begin{cases} \{j_1\} & \text{if } p > 3/4 \\ \{j_2\} & \text{if } p < 3/4 \\ \Delta(J) & \text{if } p = 3/4 \end{cases}$$

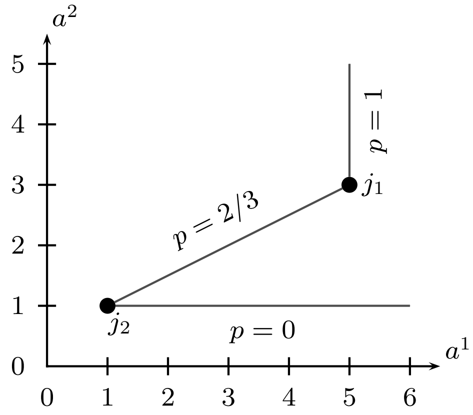


Unique equilibrium, non-revealing (Example 3)

	j_1	j_2	
k_1	$5,2$	$1,0$	p
k_2	$3,0$	$1,4$	$(1-p)$

25/

$$Y(p) = \begin{cases} \{j_1\} & \text{if } p > 2/3, \\ \{j_2\} & \text{if } p < 2/3, \\ \Delta(J) & \text{if } p = 2/3 \end{cases}$$

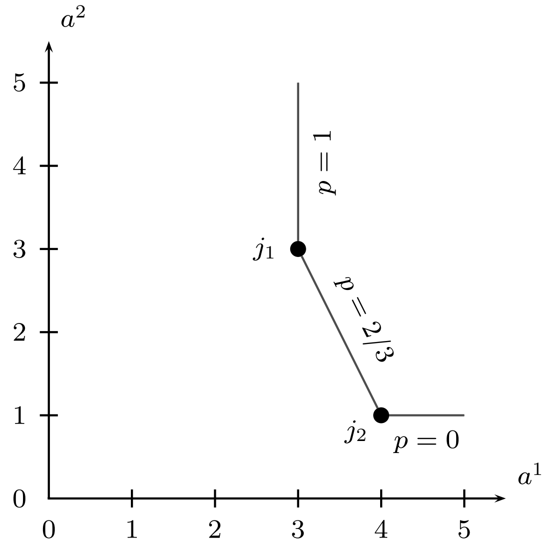


Unique equilibrium, non-revealing (Example 4)

	j_1	j_2	
k_1	$3,2$	$4,0$	p
k_2	$3,0$	$1,4$	$(1-p)$

26/

$$Y(p) = \begin{cases} \{j_1\} & \text{if } p > 2/3, \\ \{j_2\} & \text{if } p < 2/3, \\ \Delta(J) & \text{if } p = 2/3 \end{cases}$$



Partial revelation of information: Example 6

	j_1	j_2	j_3	j_4	
k_1	$4, 0$	$2, 7$	$5, 9$	$1, 10$	p
k_2	$1, 10$	$4, 7$	$4, 4$	$2, 0$	$1 - p$

27/

$$Y(p) = \begin{cases} \{j_1\} & \text{if } p < 3/10 \\ \Delta(\{j_1, j_2\}) & \text{if } p = 3/10 \\ \{j_2\} & \text{if } p \in (3/10, 3/5) \\ \Delta(\{j_2, j_3\}) & \text{if } p = 3/5 \\ \{j_3\} & \text{if } p \in (3/5, 4/5) \\ \Delta(\{j_3, j_4\}) & \text{if } p = 4/5 \\ \{j_4\} & \text{if } p > 4/5 \end{cases}$$

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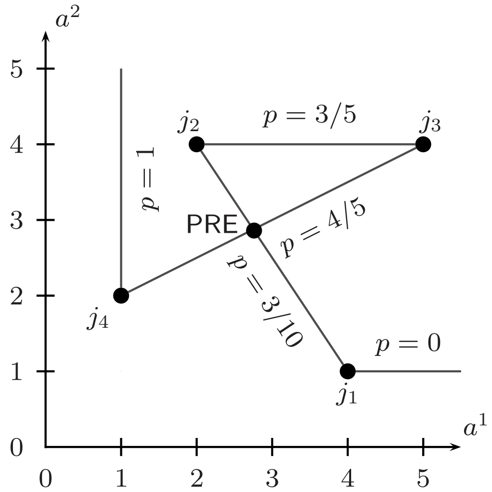


✍ Characterize explicitly players' strategies inducing the PRE when $p = 1/2$

Monotonic Games

Grossman (1981); Grossman and Hart (1980); Milgrom (1981);
Milgrom and Roberts (1986); Watson (1996),...

$$A^k(j) > A^k(j') \Leftrightarrow j > j', \quad \forall\, k \in K$$

Examples:

29/
- A seller who wants to maximize sells
- A manager who wants to maximize the value of the firm
- A worker who wants the job with the highest wage (whatever his competence)
- A firm who wants its competitors to decrease their productions

Theorem (Monotonic games) *In a monotonic cheap talk games, every Nash equilibrium in which the decision maker uses pure strategies is non-revealing*

Proof. ✍ □

In particular, if $\arg\max_{j \in J} B^k(j)$ is unique for every k and depends on k, then there is no fully revealing equilibrium

But information transmission is still possible in monotonic games

- A fully revealing equilibrium may exist if the DM uses mixed strategies (Example 7)
- Even if $\arg\max_{j \in J} B^k(j)$ is unique for every k, a partially revealing equilibrium may exist (Example 8)

30/
- If the DM also has private information (incomplete information on both sides), a fully revealing equilibrium in pure strategy may exist
- If information is certifiable, then a fully revealing equilibrium always exists in monotonic games
- A FRE is also possible with public cheap talk to two decisionmakers, even if the private communication games are monotonic and have a unique non-revealing equilibrium

Example 7. The following monotonic game has a FRE:

$$\sigma(k_1) = a \quad \sigma(k_2) = b$$

$$\tau(a) = \frac{2}{3}\, j_3 + \frac{1}{3}\, j_5 \quad \tau(b) = \frac{1}{6}\, j_2 + \frac{5}{6}\, j_4$$

31/

	j_1	j_2	j_3	j_4	j_5
k_1	$1, 2$	$2, 0$	$3, 3$	$4, 0$	$5, 3$
k_2	$1, 2$	$2, 3$	$3, 0$	$4, 3$	$5, 0$

Example 8. The following monotonic game has a PRE when $\Pr[k_1] = 3/10$:

$$\sigma(k_1) = \frac{1}{3}\, a + \frac{2}{3}\, b \quad \sigma(k_2) = \frac{4}{7}\, a + \frac{3}{7}\, b$$

$$\tau(a) = \frac{1}{3}\, j_1 + \frac{2}{3}\, j_3 \quad \tau(b) = \frac{2}{3}\, j_2 + \frac{1}{3}\, j_3$$

32/

	j_1	j_2	j_3
k_1	$1, 7$	$2, 0$	$3, 4$
k_2	$1, 7$	$2, 10$	$3, 9$

Incomplete information on both sides: type $l \in L$ for the DM (private signal)

➥ Prior probability distribution $p \in \Delta(K \times L)$

Example 9. The following monotonic game has a pure strategy FRE when $p = \begin{pmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{pmatrix}$:

33/

$$\sigma(k_1) = \mathsf{a} \quad \sigma(k_2) = \mathsf{b},$$

$$\tau(\mathsf{a}, l_1) = \tau(\mathsf{b}, l_2) = j_2 \quad \tau(\mathsf{a}, l_2) = \tau(\mathsf{b}, l_1) = j_1.$$

	l_1		l_2	
	j_1	j_2	j_1	j_2
k_1	$1,0$	$2,2$	$1,1$	$2,0$
k_2	$1,1$	$2,0$	$1,0$	$2,2$

Crawford and Sobel's (1982) Model

- Types of the expert: $T = [0,1]$, uniformly distributed
- Cheap talk messages of the expert: $M = [0,1]$
- Actions of the decisionmaker: $A = [0,1]$
- 34/ Utility of the expert (player 1): $u_1(a;t) = -\big[a - (t+b)\big]^2, \quad b > 0$
- Utility of the decisionmaker (player 2): $u_2(a;t) = -\big[a - t\big]^2$

Both players' preferences depend on the state: when t increases, both players want the action to increase but the ideal action of the expert, $a_1^*(t) = t + b$, is always higher than the ideal action of the decisionmaker, $a_2^*(t) = t$

Applications:

- Relationship between a doctor and his patient, where the patient has a bias towards excessive medication
- Choice of expenditure on a public project
- Choice of departure time for two friends (with different risk attitude) to take a plane (one having private information about flight time)
- Hierarchical relationships in organizations (e.g., choice = effort level)

"n-partitional" equilibria, in which n different messages are sent:

$$\sigma_1(t) = \begin{cases} m_1 & \text{if } t \in [0, x_1) \\ \vdots & \vdots \\ m_k & \text{if } t \in [x_{k-1}, x_k) \\ \vdots & \vdots \\ m_n & \text{if } t \in [x_{n-1}, 1] \end{cases}$$

where $0 < x_1 < \cdots < x_{n-1} < x_n = 1$ and $m_k \neq m_l \ \forall\, k \neq l$

and $n \leq n^*(b) =$ maximal number of different messages that can be sent in equilibrium, decreasing with b

$$\Rightarrow \ \sigma_2(m_k) = \mathrm{E}(\boldsymbol{t} \mid m_k) = \mathrm{E}\left(\boldsymbol{t} \mid t \in [x_{k-1}, x_k)\right) = \frac{x_{k-1} + x_k}{2}$$

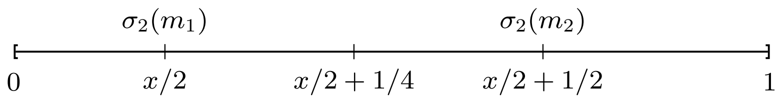
Equilibrium conditions for $n = 2$

$$\sigma_1(t) = \begin{cases} m_1 & \text{if } t \in [0, x) \\ m_2 & \text{if } t \in [x, 1] \end{cases} \Rightarrow \sigma_2(m) = \begin{cases} x/2 & \text{if } m = m_1 \\ (x+1)/2 & \text{if } m = m_2 \end{cases}$$

For off the equilibrium path messages $m \notin \{m_1, m_2\}$, it suffices to consider the
37/ same beliefs as along the equilibrium path

Example: $m_1 = 0$, $m_2 = 1$ and $\mu(t \mid m) \sim \begin{cases} \mathcal{U}[0, x] & \text{if } m \in [0, x) \\ \mathcal{U}[x, 1] & \text{if } m \in [x, 1] \end{cases}$

Given the decisionmaker's strategy σ_2, the expert of type t will send the message $m \in \{m_1, m_2\}$ which induces the closest action to $t + b$



so $\sigma_1(t) = \begin{cases} m_1 & \text{if } \; t + b < x/2 + 1/4 \\ m_2 & \text{if } \; t + b > x/2 + 1/4 \end{cases}$

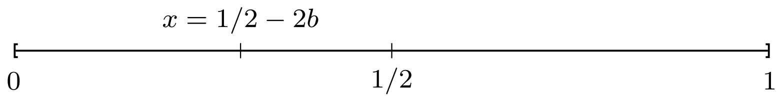
38/ We started from

$$\sigma_1(t) = \begin{cases} m_1 & \text{if } \; t < x \\ m_2 & \text{if } \; t \geq x \end{cases} = \begin{cases} m_1 & \text{if } \; t + b < x + b \\ m_2 & \text{if } \; t + b \geq x + b \end{cases}$$

so we must have $x + b = x/2 + 1/4 \Leftrightarrow \boxed{x = 1/2 - 2\,b}$

➠ There is a 2-partitional equilibrium if and only if $b \leq 1/4$

➥ The interval $[x, 1]$ is $4\,b$ larger than $[0, x]$



This can be generalized to n-partitional equilibria:

39/ For every k, the sender of type $t = x_k$ should be indifferent between sending m_k and m_{k+1}

$\Rightarrow$ his ideal point, $x_k + b$, should be in the middle of $\frac{x_{k-1}+x_k}{2}$ and $\frac{x_k+x_{k+1}}{2}$

$$\Rightarrow \; x_k + b = \frac{\frac{x_{k-1}+x_k}{2} + \frac{x_k+x_{k+1}}{2}}{2} = \frac{x_{k-1} + 2\,x_k + x_{k+1}}{4}$$

so $[x_{k+1} - x_k] = [x_k - x_{k-1}] + 4\,b$

$$\Rightarrow \; x_k = x_1 + (x_1 + 4b) + (x_1 + 2\,(4b)) + \cdots + (x_1 + (k-1)\,(4b))$$
$$= k\,x_1 + (1 + 2 + \cdots + (k-1))\,4b = k\,x_1 + \frac{k(k-1)}{2}\,4b$$

In particular, $1 = x_n = n\,x_1 + n(n-1)\,2b$

$$\Rightarrow \boxed{x_1 = 1/n - 2(n-1)b} \Rightarrow \boxed{x_k = k/n - 2kb(n-k)}$$

☞ A n-partitional equilibrium exists if $b < \frac{1}{2n(n-1)}$

40/ ☞ Given b, the largest n such that there exists a n-partitional equilibrium is the largest n, denoted by $n^*(b)$, such that

$$2n(n-1)b < 1 \quad \Leftrightarrow \quad n^2 - n - 1/2b < 0$$
$$\Leftrightarrow \quad n < \frac{1 + \sqrt{1 + 2/b}}{2} \quad = \quad \begin{cases} 2 & \text{if } b = 1/4 \\ +\infty & \text{if } b \to 0 \end{cases}$$

but full revelation of information is impossible as long as players' preferences are not perfectly aligned ($b \neq 0$)

✍ For which positive values of b does there exist a 3-partitional equilibrium?

✍ Characterize all equilibria when $b = 1/10$

✍ Verify that, in general, the best equilibrium for the expert depends on his type

Welfare comparison of equilibria

Ex-ante expected utility of the decisionmaker at a n-partitional equilibrium:

41/

$$\begin{aligned} EU_2 &= \mathrm{E}\left[-[\sigma_2(\sigma_1(t)) - t]^2\right] = -\int_0^1 [\sigma_2(\sigma_1(t)) - t]^2\, dt \\ &= -\sum_{k=1}^n \int_{x_{k-1}}^{x_k} [\sigma_2(m_k) - t]^2\, dt = -\sum_{k=1}^n \int_{x_{k-1}}^{x_k} \left[\frac{x_{k-1}+x_k}{2} - t\right]^2 dt \\ &= -\sum_{k=1}^n \frac{1}{3}\left[\left(t - \frac{x_{k-1}+x_k}{2}\right)^3\right]_{x_{k-1}}^{x_k} = -\frac{1}{12}\sum_{k=1}^n (x_k - x_{k-1})^3 \end{aligned}$$

$$\begin{aligned} x_k - x_{k-1} &= k/n - 2k\,b\,(n-k) - ((k-1)/n - 2(k-1)\,b\,(n-(k-1))) \\ &= 1/n + 2b\,(2k-n-1) \end{aligned}$$

so $EU_2 = -\frac{1}{12}\sum_{k=1}^n \big(1/n + \underbrace{2b(2k-n-1)}_{\alpha}\big)^3$

In α, members in k cancel out with members in $n-k+1$, so

$$EU_2 = -\frac{1}{12}\sum_{k=1}^n \big(1/n^3 + \underbrace{3\alpha/n^2}_{0} + 3\alpha^2/n + \underbrace{\alpha^3}_{0}\big) = -\frac{1}{12n^2} - \frac{1}{4n}\sum_{k=1}^n \alpha^2$$

42/

After some simplifications, using $\sum_1^n k^2 = \frac{n(n+1)(2n+1)}{6}$, we get

$$EU_2 = -\frac{1}{12n^2} - \frac{b^2(n^2-1)}{3}$$

➠ With a fixed n, the expected payoff of the decisionmaker decreases with b

Ex-ante expected utility of the expert at a n-partitional equilibrium:

43/

$$\begin{aligned} EU_1 &= \mathrm{E}\left[-[\sigma_2(\sigma_1(t)) - t - b]^2\right] \\ &= -\sum_{k=1}^{n} \int_{x_{k-1}}^{x_k} \left[\frac{x_{k-1}+x_k}{2} - t - b\right]^2 dt \\ &= -\sum_{k=1}^{n} \Big(\int_{x_{k-1}}^{x_k} \left[\frac{x_{k-1}+x_k}{2} - t\right]^2 dt + \int_{x_{k-1}}^{x_k} b^2\, dt \\ & \underbrace{-2b \int_{x_{k-1}}^{x_k} \left[\frac{x_{k-1}+x_k}{2} - t\right] dt \Big)}_{0} \end{aligned}$$

so $EU_1 = EU_2 - b^2$ is also decreasing with b when n is fixed

Which equilibrium is the most efficient?

➥ We compare EU_2 (or EU_1) at a n-partitional equilibrium with EU_2 (or EU_1) at a $(n-1)$-partitional equilibrium:

After some simplifications we find, for every $n \geq 1$, $EU_2[n] - EU_2[n-1] > 0$ if and only if

$$b < \frac{1}{2n(n-1)}$$

which is exactly the existence condition for a n-partitional equilibrium

44/

Remark If information could be transmitted credibly, then the expected payoffs of both players would be higher than in all equilibria since we would have $EU_2 = 0$ and $EU_1 = -b^2$. We will see that the same outcome is achieved with certifiable information

Generalization

All equilibria are partitional equilibria, and n-partitional equilibria exist for increasing values of n when players' conflict of interest decrease, in a larger class of games:

- Types of the expert: T, distribution $F(t)$ with density $f(t)$
- Cheap talk messages $M = [0,1]$ and actions $A = \mathbb{R}$
- Utility of the expert (decisionmaker, resp.): $u_1(a;t)$ ($u_2(a;t)$, resp.)

Assumptions: for every $i = 1$, 2 and $t \in T$

45/

(i) u_i is twice continuously differentiable

(ii) For all $t \in T$, there exists $a \in \mathbb{R}$ such that $\partial u_i / \partial a = 0$

(iii) $\partial^2 u_i / \partial a^2 < 0 \Rightarrow u_i$ has a unique maximum $a_i^*(t)$

(iv) $\partial^2 u_i / \partial a \partial t > 0 \Rightarrow$ the ideal action $a_i^*(t)$ is strictly increasing with t

(v) $a_1^*(t) \neq a_2^*(t)$ for all $t \in T$

In general, equilibria cannot be compared in terms of efficiency anymore

Variations and Extensions

- **Burned Money.**

In general, in standard signaling games, information revelation stems from the dependence between signaling costs and the sender's type

For example, in the labor market signaling game of Spence, if the cost of education is the same for different abilities of the worker, then information revelation would be

46/ impossible

But this is not general. In Example 3, if cost(a) = 3 $\forall\, k$ then a FRE exists ($k_1 \to a$ and $k_2 \to b$) while cheap talk is not credible

In this example, strategic money burning improves Pareto efficiency. The same phenomenon is possible in Crawford and Sobel's model (see Austen-Smith and Banks, 2000, 2002)

- **Cheap Talk vs. Delegation**

Consider again the model of Crawford and Sobel (1982):

- Expert (player 1): $u_1(a;t) = -\big[a - (t+b)\big]^2, \quad b > 0$
- Decisionmaker (player 2): $u_2(a;t) = -\big[a - t\big]^2$

47/ Alternative to communication: the decisionmaker delegates the decision $a \in [0,1]$ to the expert

Example: in a firm, instead of collecting all the information from the different hierarchical levels of the organization, a manager may delegate some decisions (e.g., investment decisions) to agents in lower levels of the hierarchies, even if these agents do not have exactly same incentives as the manager

Delegation of the decision to the expert $\Rightarrow$ action $a_1^*(t) = t + b$ is chosen when the expert's type is t

$$\Rightarrow \quad \forall\, t \quad \begin{cases} EU_1^D = u_1(a_1^*(t);t) = 0 \\ EU_2^D = u_2(a_1^*(t);t) = -b^2 \end{cases}$$

In the cheap talk game:

48/

$$\begin{cases} EU_1 = EU_2 - b^2 \\ EU_2 = -\frac{1}{12n^2} - \frac{b^2(n^2-1)}{3} \end{cases}$$

where n is such that $b \leq \frac{1}{2n(n-1)}$

Of course, the expert always prefers delegation. The DM prefers delegation to cheap talk if $EU_2^D \geq EU_2 \Leftrightarrow b^2 \leq \frac{1}{12n^2} + \frac{b^2(n^2-1)}{3}$

Hence, delegation is optimal if $b \leq 1/4$ and $n \geq 2$ or $b \leq 1/\sqrt{12} \simeq 1/3.5$ and $n = 1$

In particular, delegation is optimal whenever there is an informative partitional equilibrium in the cheap talk game (i.e., $b \leq 1/4$)

With an extreme bias ($b > 1/3.5$) the decision maker plays the optimal action of the silent game $a = E[\boldsymbol{t}] = 1/2$ (no delegation, no informative communication)

49/ $\Rightarrow$ Delegation of the decision right is often preferred over cheap talk because the welfare loss caused by self-interested communication is higher than costs of biased decision-making

Dessein (2002) shows more generally (for a non-uniform prior distribution) that delegation is better than communication, except when the expert has a small informational advantage and communication is very noisy

- **Cheap Talk vs. Commitment.**

Consider a mechanism design / principal-agent approach, but without transfers (Melumad and Shibano, 1991)

The decisionmaker (the principal) commits to a decision rule

$$a : T \to A$$

that maximizes his utility under the agent's informational incentive constraint

50/ (w.l.o.g. by the revelation principle)

$$\max_{a(\cdot)} - \int_0^1 (a(t) - t)^2 \; dt$$
$$u.t.c. \; -(a(t) - t - b)^2 \geq -(a(t') - t - b)^2, \quad \forall \, t, t' \in T.$$

Of course, if $b \neq 0$, the (first best) decision rule $a(t) = t$ does not satisfy the informational incentive constraint

The informational incentive constraint implies

$$a'(t)(a(t) - t - b) = 0, \quad \forall\, t \in T,$$

so on every interval $a(t)$ is either constant or $a(t) = t + b = a_1^*(t)$. In particular, full separation, $a(t) = t + b$, and full bunching, $a(t) = a$, satisfy the constraint

Assuming continuity, the decision rule should take the following form, with $0 \leq t_1 \leq t_2 \leq 1$:

51/

$$a(t) = \begin{cases} t_1 + b & \text{if } t \leq t_1, \\ t + b & \text{if } t \in [t_1, t_2], \\ t_2 + b & \text{if } t \geq t_2, \end{cases}$$

or should be constant on T

Hence, the principal minimizes

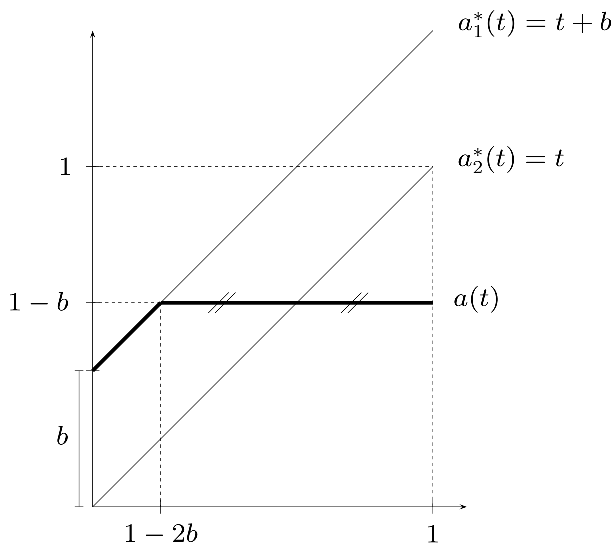
$$\int_0^{t_1} (\underbrace{a(t)}_{t_1+b} - t)^2 \; dt + \int_{t_1}^{t_2} (\underbrace{a(t)}_{t+b} - t)^2 \; dt + \int_{t_2}^{1} (\underbrace{a(t)}_{t_2+b} - t)^2 \; dt$$
$$= -(1/3)(b^3 - (t_1 + b)^3) + b^2(t_2 - t_1) - (1/3)((t_2 + b - 1)^3 - b^3),$$

if $0 \leq t_1 \leq t_2 \leq 1$, or chooses $a(t) = 1/2$ for all t

52/

The solution is $(t_1, t_2) = (0, 1 - 2b)$ if $b \leq 1/2$, and $a(t) = 1/2$ for all $t \in T$ if $b \geq 1/2$

53/



Comparing cheap talk, delegation (D) and commitment (C), we have:

$$EU_1^D \geq EU_1^C \geq EU_1,$$
$$EU_2^C \geq EU_2^D \geq EU_2$$

$\Rightarrow$ The best situation for the decisionmaker is commitment (contracting) and that of the expert, delegation. Whatever the equilibrium, cheap talk communication is always worse than delegation and commitment for both players

54/ **Remark.** The optimal mechanism can be implemented with a *delegation set* $\mathcal{D} = [0, 1 - b]$, the principal letting the agent choose any action in $\mathcal{D}$

- **Multiple Senders and Multidimensional Cheap Talk**

Usual models of cheap talk: *unidimensional* policy decision and information

Basic insight: information transmission decreases when the conflict of interest between the interested parties (the senders) and the decisionmaker (the receiver) increases

Battaglini (2002): Not true in a multidimensional environment, in which a fully revealing equilibrium may exist even when the conflict of interest is arbitrary large

55/

Model:

- *State* $\theta \in \Theta = \mathbb{R}^d$
- *Policy* $x \in \mathbb{R}^d$
- *Two* perfectly informed *experts*, $i = 1, 2$
- The *policy maker*, p, is uninformed

For all $i \in \{1, 2, p\}$, $u_i(x, \theta)$ is continuous and quasi concave in x

Ideal points: $\theta + x_i \in \mathbb{R}^d$, where $x_p = 0$

Assume quadratic utilities:

$$u_i(x, \theta) = -\sum_{j=1}^{d} (x^j - (x_i^j + \theta))^2$$

Timing:

① Nature chooses θ

② Experts simultaneously send a message about θ to the DM

56/

③ The DM chooses x

Expert i's strategy: $s_i : \Theta \to \mathcal{M}$

DM's belief: $\mu : \mathcal{M} \times \mathcal{M} \to \Delta(\Theta)$

DM's strategy: $x : \mathcal{M} \times \mathcal{M} \to \mathbb{R}^d$

Fully Revealing Equilibrium (FRE):

$$\mu(\theta \mid s_1(\theta), s_2(\theta)) = 1, \quad \text{for all } \theta \in \Theta$$

Unidimensional Case.

- Gilligan and Krehbiel (1989, American Journal of Political Science)
- Krishna and Morgan (2001, QJE)
- Battaglini (2002, Ecta)

A FRE may exist if experts' ideal points are not too extreme

- E.g., when $x_1, x_2 > 0$, there is a FRE $s_1(\theta) = s_2(\theta) = \theta$ with

$$x(s_1(\theta), s_2(\theta)) = \min\{s_1(\theta), s_2(\theta)\}$$

- When $x_1 < 0 < x_2$, a FRE exists if $|x_1| + |x_2|$ is not too large, but may rely on implausible (extreme) beliefs off the equilibrium path

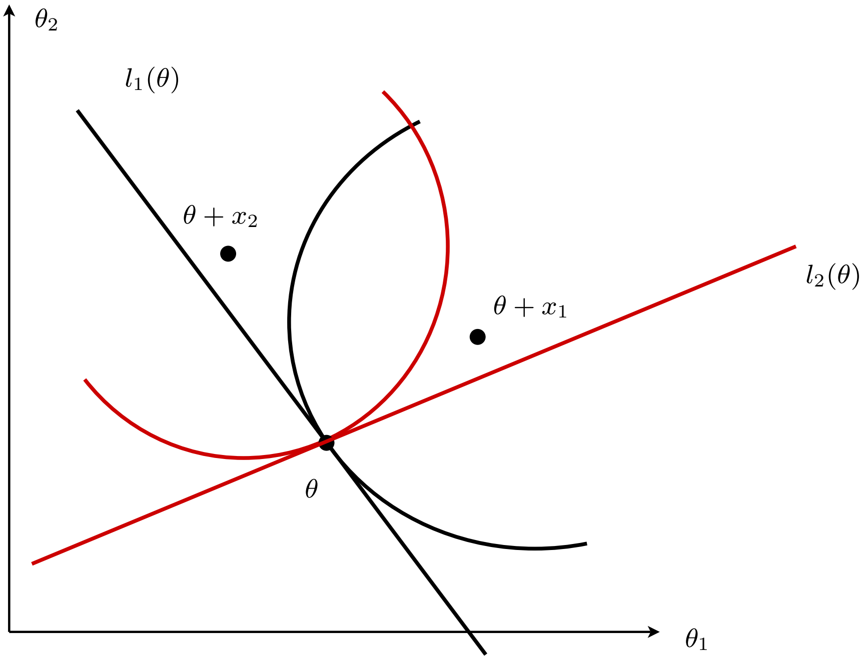
Multidimensional Case.

Proposition 1 (Battaglini, 2002) *If $d = 2$ and $x_1 \neq \alpha x_2$ for all $\alpha \in \mathbb{R}$ (i.e., x_1 and x_2 and linearly independent), then there is a FRE*

Proof.

Each expert will reveal the tangent of the other expert's indifference curve at the DM's ideal point θ

59/



Let

$$l_i(\theta)$$

be the tangent of i's indifference curve at the DM's ideal point θ

By linear independence, these tangents cross only once, so $l_1(\theta) \cap l_2(\theta) = \theta$

The following strategy profile and beliefs constitute a FR PBE:

- $s_i(\theta) = l_j(\theta)$, $i \neq j$
60/
- $\mu(s_1, s_2) = s_1 \cap s_2$ (and any point in $l_i(\theta)$ if $s_i \cap l_i(\theta) = \emptyset$)
- $x(s_1, s_2) = \mu(s_1, s_2)$

If expert i reveals $\hat{s}_i$ when the state is θ, then the action of the DM is

$$x(\hat{s}_i, s_j(\theta)) = \mu(\hat{s}_i, l_i(\theta)) \in l_i(\theta)$$

which, by construction, is the closest to i's ideal point when $\hat{s}_i = l_j(\theta)$ □

Remark The result can be extended to more than two dimensions of the policy space and to quasi-concave utilities (not necessarily quadratic), but may not be robust to the timing of the game (sequential cheap talk)

• **Lobbying with Several Audiences.** Farrell and Gibbons (1989) show in a model with two decisionmakers that the expert's announcement may be more credible when communication takes place publicly

Example:

	Q		R	
	q_1	q_2	r_1	r_2
k_1	v_1, x_1	$0, 0$	w_1, y_1	$0, 0$
k_2	$0, 0$	v_2, x_2	$0, 0$	w_2, y_2

There exists a fully revealing equilibrium when the lobbyist communicates *privately* with the decisionmaker Q (R, respectively) if and only if $v_1 \geq 0$ and $v_2 \geq 0$ ($w_1 \geq 0$ and $w_2 \geq 0$, respectively)

There exists a fully revealing equilibrium when the lobbyist communicates *publicly* with the two decisionmakers if and only if $v_1 + w_1 \geq 0$ and $v_2 + w_2 \geq 0$

Mutual discipline: There is no separating equilibrium in private, but there is in public. E.g., when $v_1 = w_2 = 3$ and $v_2 = w_1 = -1$

Some Experimental Evidence

Dickhaut et al. (1995, ET).

- Crawford and Sobel (1982) with 4 states and 4 actions
- Five treatments (biases)

$$\underbrace{b_1, b_2}_{FRE,PRE,NRE}, \underbrace{b_3}_{PRE,NRE}, \underbrace{b_4, b_5}_{NRE}$$

63/

- 12 repetitions among 8 subjects with random matching

Results:

- Observed average distance between states and actions increases with the bias b
- Receivers' average payoffs decrease with b
- Two much information is revealed when it should not (b_4, b_5)

Cai and Wang (2006, GEB).

- Crawford and Sobel (1982) with 5 states and 9 actions
- Four treatments (biases) with the most informative equilibria being

$$\underbrace{b_1}_{FRE}, \underbrace{b_2}_{PRE1}, \underbrace{b_3}_{PRE2}, \underbrace{b_4}_{NRE}$$

Results:

64/

- Observed correlation between
  - states and actions
  - messages and actions
  - states and messages

  decreases with the bias b

- Receivers' and Senders' average payoffs decrease with b, and are consistent with the most informative equilibrium
- Actual strategies are not consistent with equilibrium strategies, except when $b = b_1$ (FRE)
  - Senders' strategies are more revealing than predicted
  - Receivers trust senders more than predicted

65/ **Forsythe et al. (1999, RFS).**

Seller-Buyer relationship, where the seller knows the asset quality

$\Rightarrow$ adverse selection due to asymmetric information, and only the lowest quality seller does not withdraw (Akerlof, 1970 "Lemons" problem)

The unique communication equilibrium is non-revealing (monotonic game)

Results:

- Without communication possibility, actual efficiency close to theoretical efficiency
- With cheap talk communication, the adverse selection problem is not as severe as predicted
  - efficiency is significantly higher than predicted
  - but at the expense of buyers (they overpay by relying on sellers' exaggerated claims)

66/

- With certifiable information,
  - efficiency is smaller than predicted, but higher than under cheap talk
  - no wealth transfer from buyers to sellers anymore

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