Strategic Information Transmission: Cheap Talk Games

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Outline

(November 12, 2008)

- Credible information under cheap talk: Examples
- Geometric characterization of Nash equilibrium outcomes
- Expertise with a biased interested party
- Communication in organizations: Delegation vs. cheap talk vs. commitment
- Multiple Senders and Multidimensional Cheap Talk
- Lobbying with several audiences
- Some experimental evidence

General References:

- Bolton and Dewatripont (2005, chap. 5) "Disclosure of Private Certifiable Information," in "Contract Theory"
- Farrell and Rabin (1996): "Cheap Talk," Journal of Economic Perspectives
- Forges (1994): "Non-Zero Sum Repeated Games and Information Transmission," in Essays in Game Theory: In Honor of Michael Maschler
- Koessler and Forges (2006): "Multistage Communication with and without Verifiable Types", International Journal of Game Theory
- Kreps and Sobel (1994) : "Signalling," in "Handbook of Game Theory" vol. 2
- Myerson (1991, chap. 6): "Games of communication," in "Game Theory, Analysis of Conflict"
- Sobel (2007): "Signalling Games"

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In its simplest form, a cheap talk game in a specific signaling games in which messages are costless (i.e., do not enter into players' utility functions)

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$$\begin{cases} A^k(j) = j - c(k, e) = j - e/k & \text{(worker)} \\ B^k(j) = -\big[k - j\big]^2 & \text{(employer)} \end{cases}$$

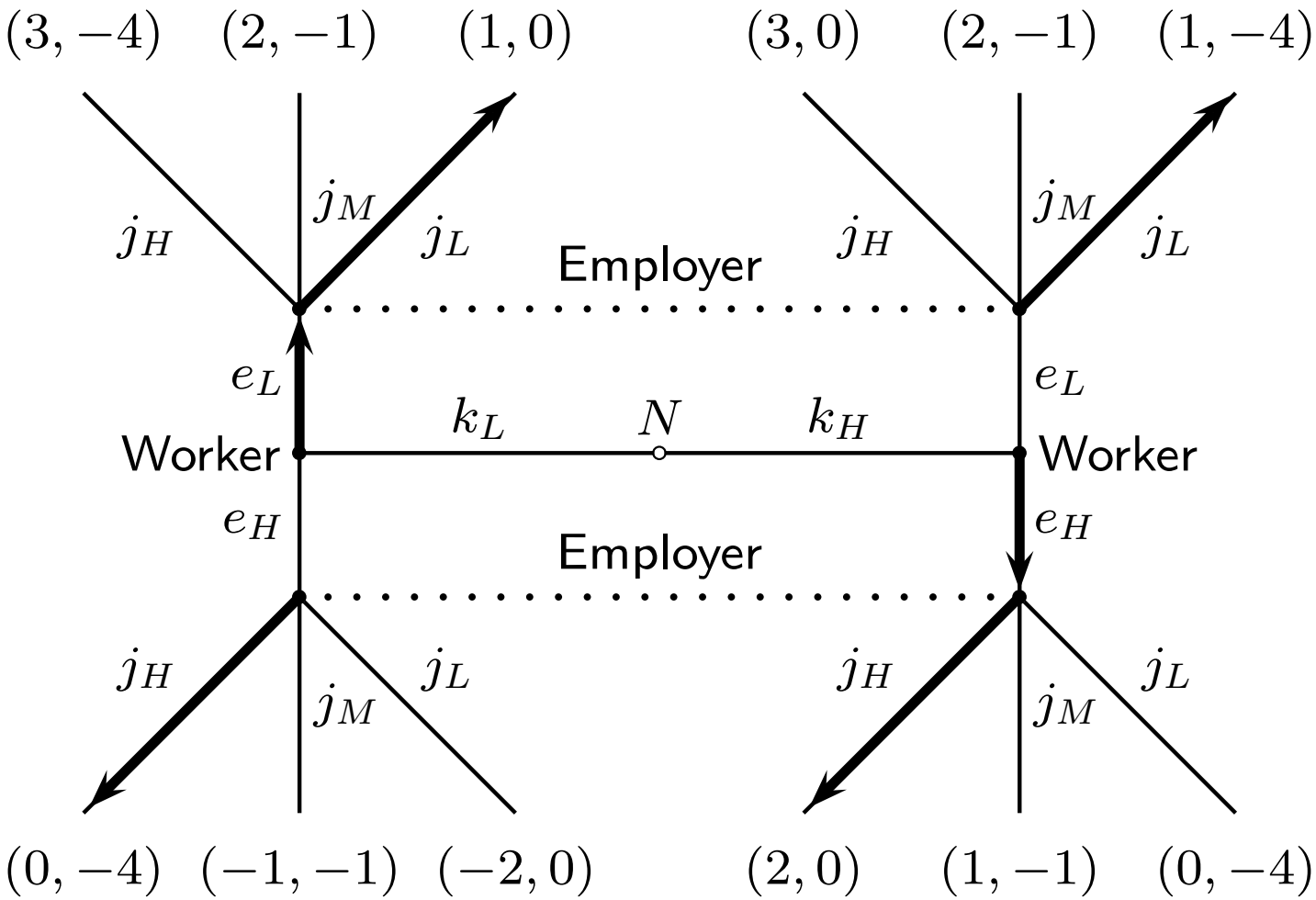

Figure 1: Fully revealing equilibrium in the labor market signaling game (example 1)

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	$j_H = 3$	$j_M = 2$	$j_L = 1$	
k_L	$3, -4$	$2, -1$	$1, 0$	$\Pr(k_L) = 1/2$

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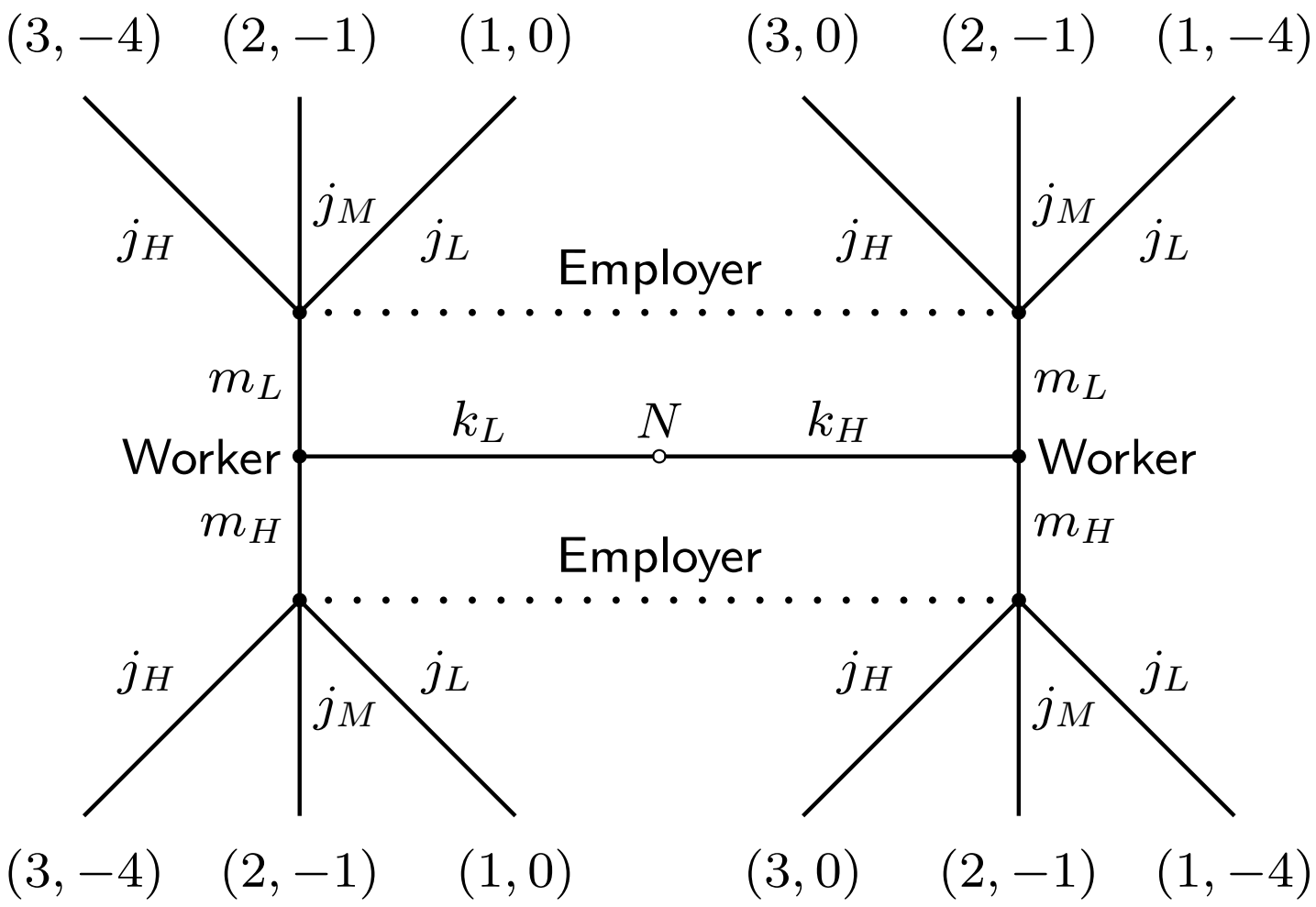

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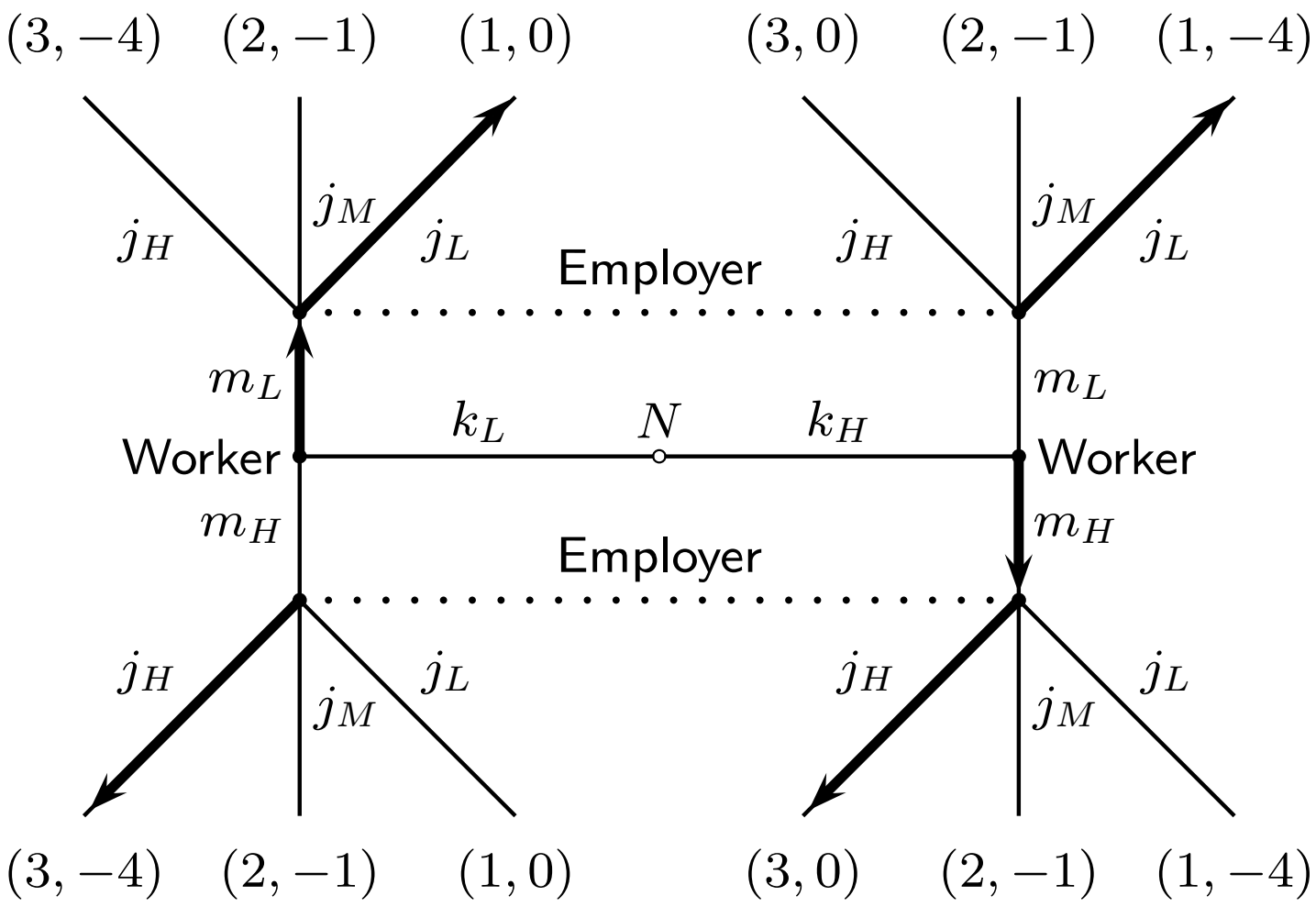

Fully revealing equilibrium?

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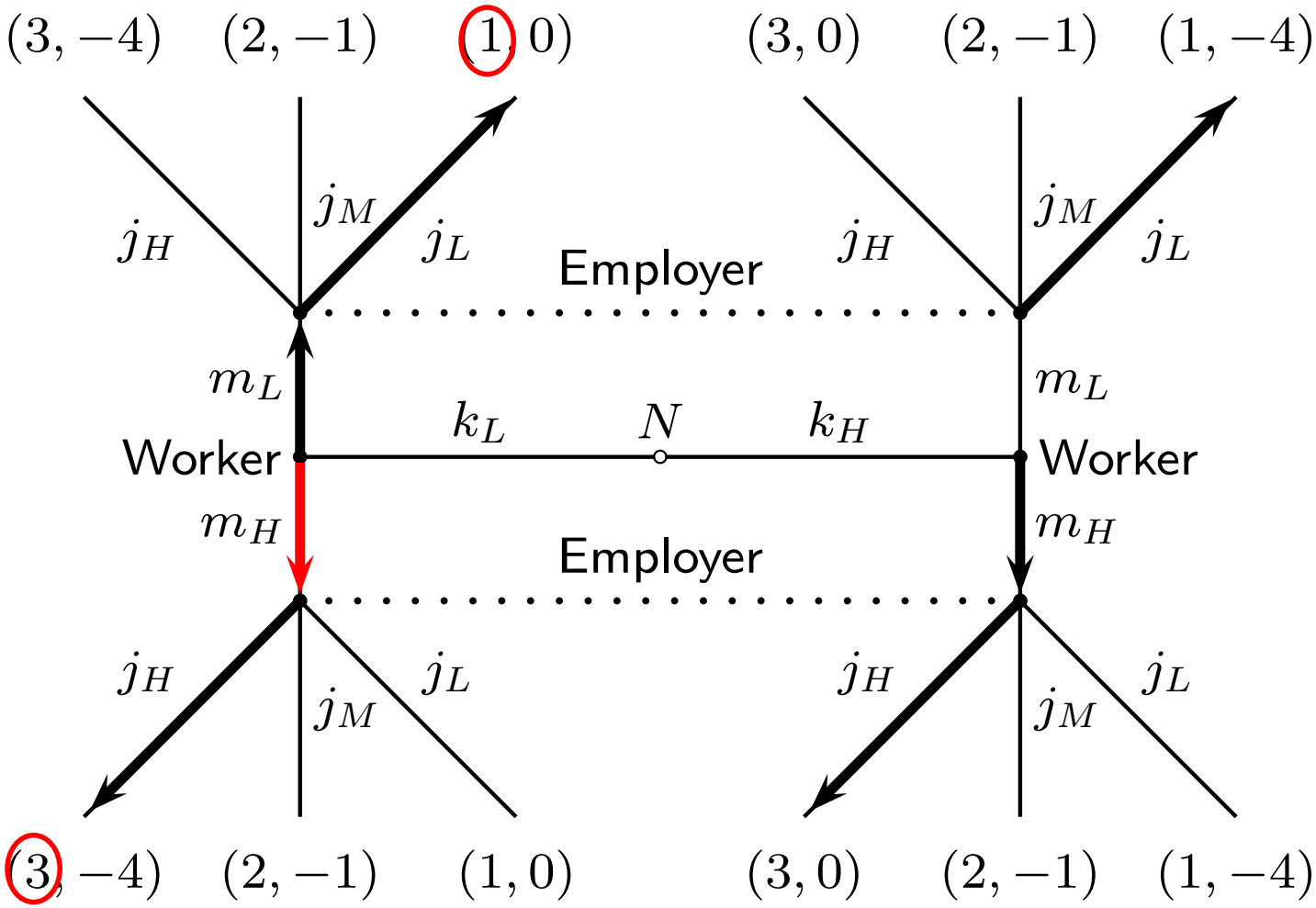

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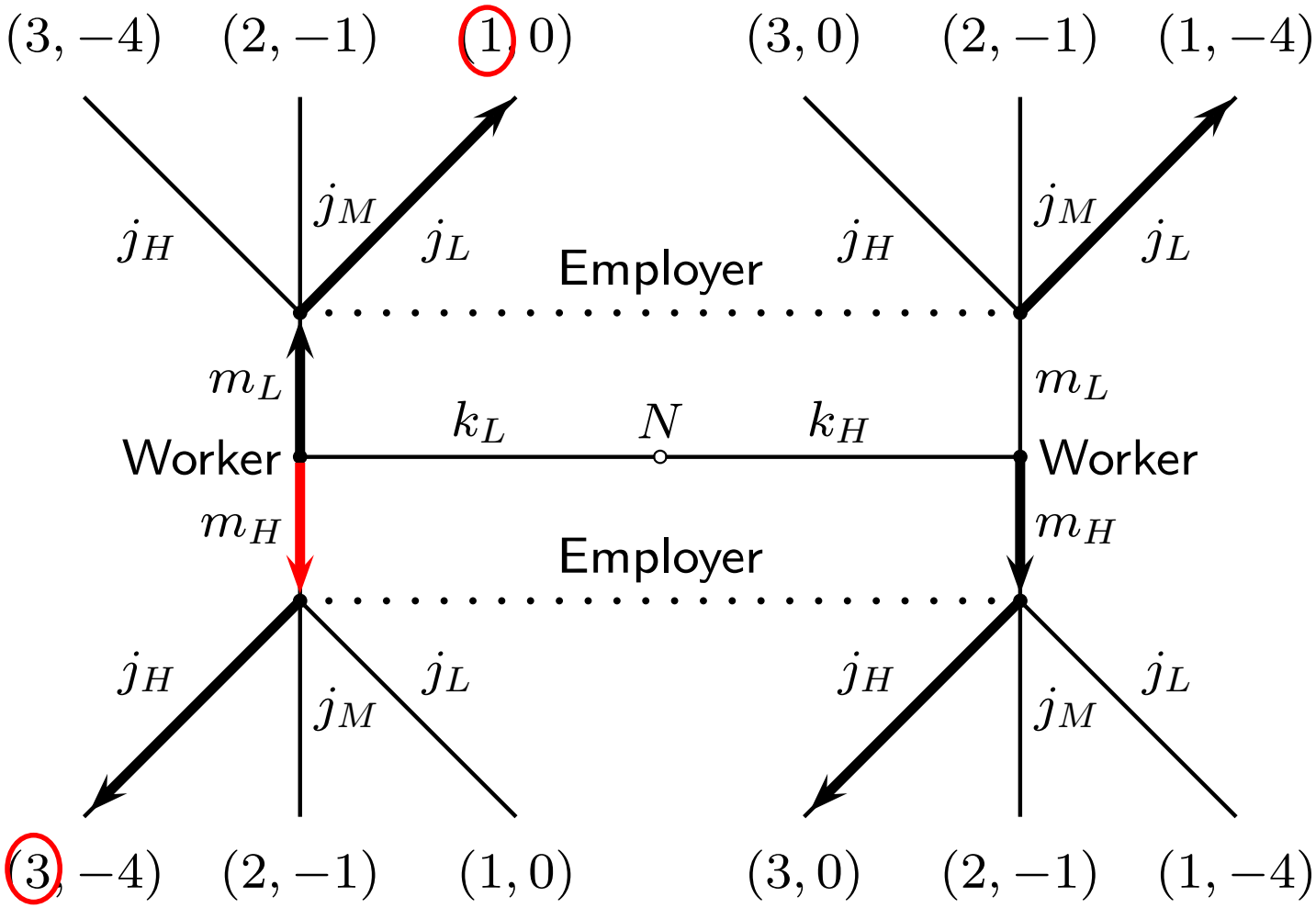

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✍ Non-revealing equilibrium? Yes, a NRE always exists in cheap talk games

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The sender's preference over the receiver's beliefs are **positively correlated** with the truth

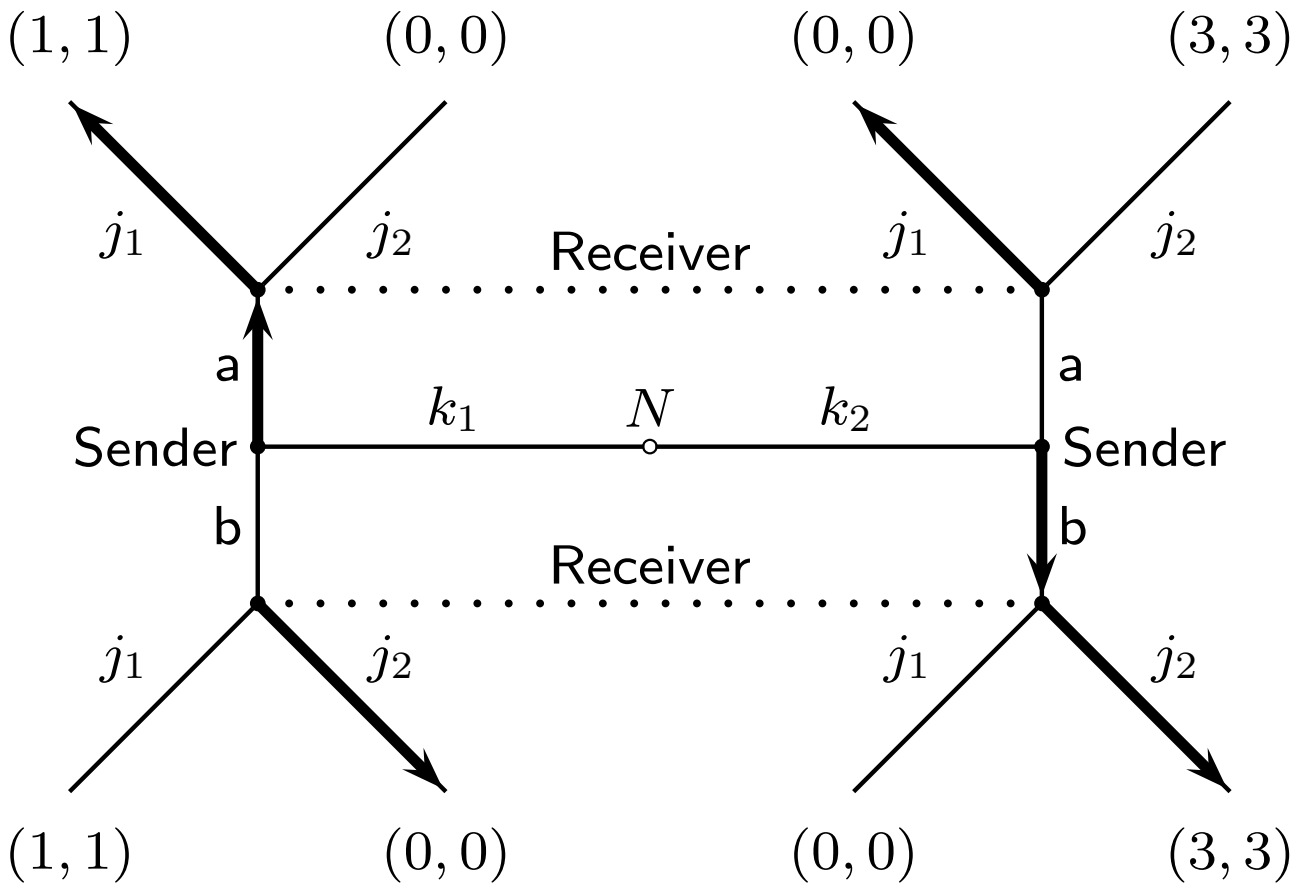

Figure 2: Fully revealing equilibrium in Example 2.

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The sender's preference over the receiver's beliefs is **not correlated** with the truth. The unique equilibrium of the cheap talk game in NR, even if when $p < 2/3$ communication of information would increase both players' payoffs

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The sender's preference over the receiver's beliefs is **negatively correlated** with the truth. The unique equilibrium of the cheap talk game in NR

Example 5. (Partial revelation of information)

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	j_1	j_2	j_3	j_4	j_5	
k_1	$1, 10$	$3, 8$	$0, 5$	$3, 0$	$1, -8$	p
k_2	$1, -8$	$3, 0$	$0, 5$	$3, 8$	$1, 10$	$1 - p$

$$Y(p) = \begin{cases} \{j_5\} & \text{if } p < 1/5 \\ \{j_4\} & \text{if } p \in (1/5, 3/8) \\ \{j_3\} & \text{if } p \in (3/8, 5/8) \\ \{j_2\} & \text{if } p \in (5/8, 4/5) \\ \{j_1\} & \text{if } p > 4/5 \end{cases}$$

Partially revealing equilibrium when $p = 1/2$:

$$\begin{cases} \sigma(k_1) = \frac{3}{4}\mathsf{a} + \frac{1}{4}\mathsf{b} \\ \sigma(k_2) = \frac{1}{4}\mathsf{a} + \frac{3}{4}\mathsf{b} \end{cases}$$

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$\Rightarrow$ equilibrium, expected utility $= \frac{3}{4}(3,8) + \frac{1}{4}(3,0) = (3,6)$ (better for the sender than the NRE and FRE)

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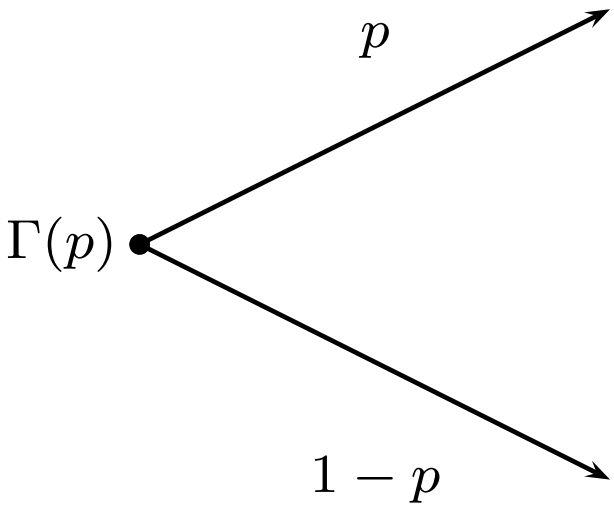
Payoffs: $A^k(j)$ and $B^k(j)$

Silent Game



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Remark Mixed actions are used in the communication extension of the game to construct equilibria in which the expert is indifferent between several messages. They also serve as punishments off the equilibrium path in communication games with certifiable information (persuasion games)

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$$\mathcal{E}(p) \equiv \{(a, \beta) : \exists\, y \in Y(p),\ a = A(y),\ \beta = p\, B^1(y) + (1-p)\, B^2(y)\}$$

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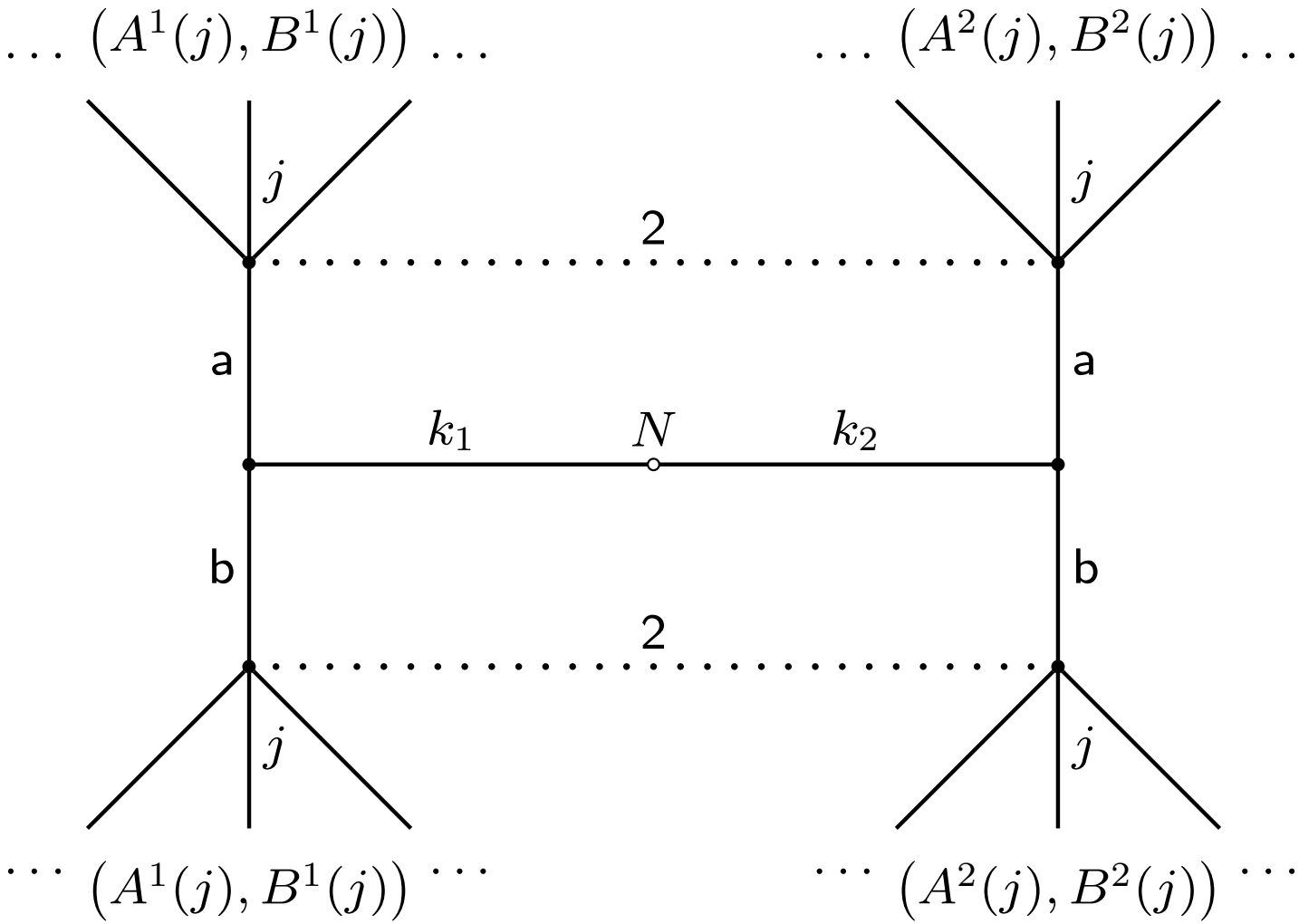

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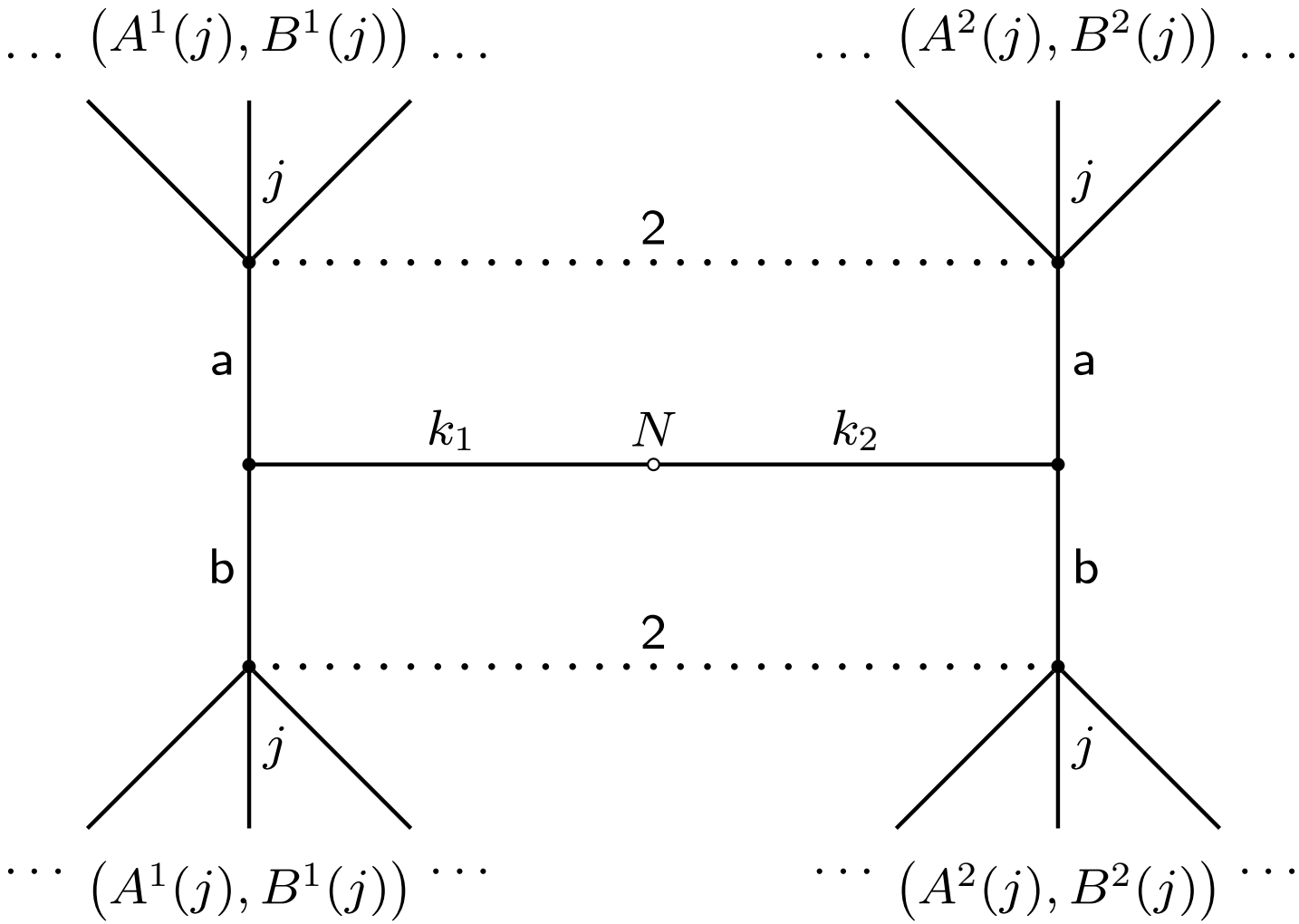

$\mathcal{E}_S^0(p)$: Equilibrium payoffs of $\Gamma_S^0(p)$

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Characterization of NE Payoffs of $\Gamma_S^0(p)$

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$$\operatorname{gr} \mathcal{E}^+ \equiv \{(a, \beta, p) \in \mathbb{R}^2 \times \mathbb{R} \times [0,1] : (a, \beta) \in \mathcal{E}^+(p)\}$$

Hart (1985, MOR), Aumann and Hart (2003, Ecta): Without any assumption on the utility functions, all equilibrium payoffs of the unilateral communication game $\Gamma_S^0(p)$ can be geometrically characterized only from the graph of the equilibrium payoff correspondence of the silent game

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Theorem (Characterization of $\mathcal{E}_S^0(p)$) *Let $p \in (0,1)$. A payoff profile (a,β) is a Nash equilibrium payoff of the unilateral communication game $\Gamma_S^0(p)$ if and only if (a,β,p) belongs to $\mathrm{conv}_a(\mathrm{gr}\,\mathcal{E}^+)$, the set points obtained by convexification of the set $\mathrm{gr}\,\mathcal{E}^+$ in (β,p) by keeping the expert's payoff, a, constant:*

$$\mathcal{E}_S^0(p) = \{(a,\beta) \in \mathbb{R}^2 \times \mathbb{R} : (a,\beta,p) \in \mathrm{conv}_a(\mathrm{gr}\,\mathcal{E}^+)\}$$

Illustrations

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Unique equilibrium, non revealing (Example 1)

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Optimal decisions in the silent game:

$$Y(p) = \begin{cases} \{j_H\} & \text{if } p < 1/4 \\ \Delta(\{j_H, j_M\}) & \text{if } p = 1/4 \\ \{j_M\} & \text{if } p \in (1/4, 3/4) \\ \Delta(\{j_M, j_L\}) & \text{if } p = 3/4 \\ \{j_L\} & \text{if } p > 3/4 \end{cases}$$

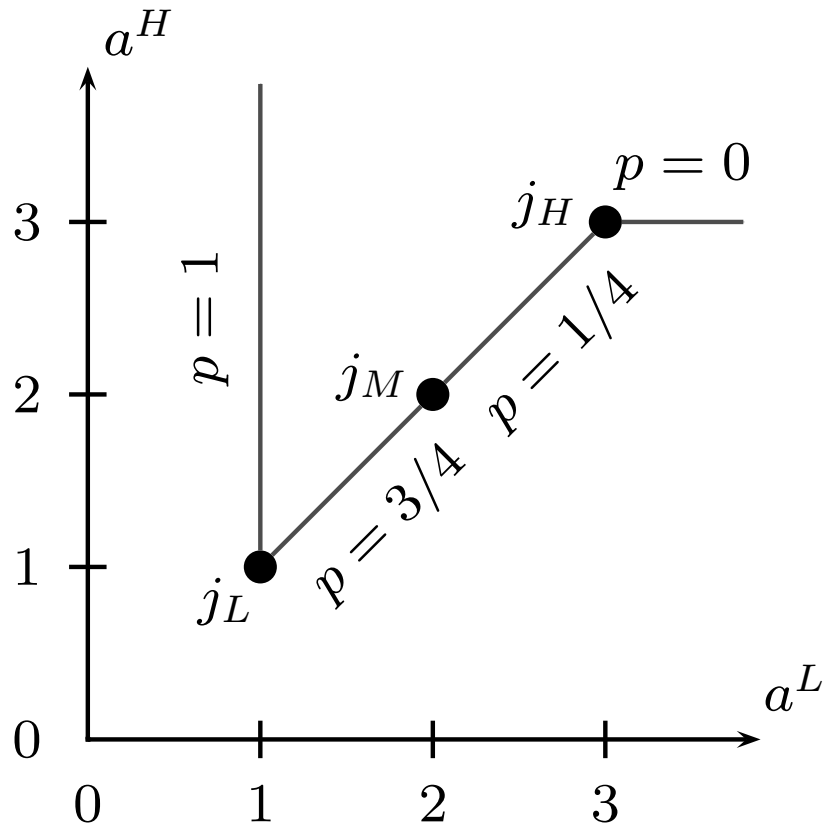
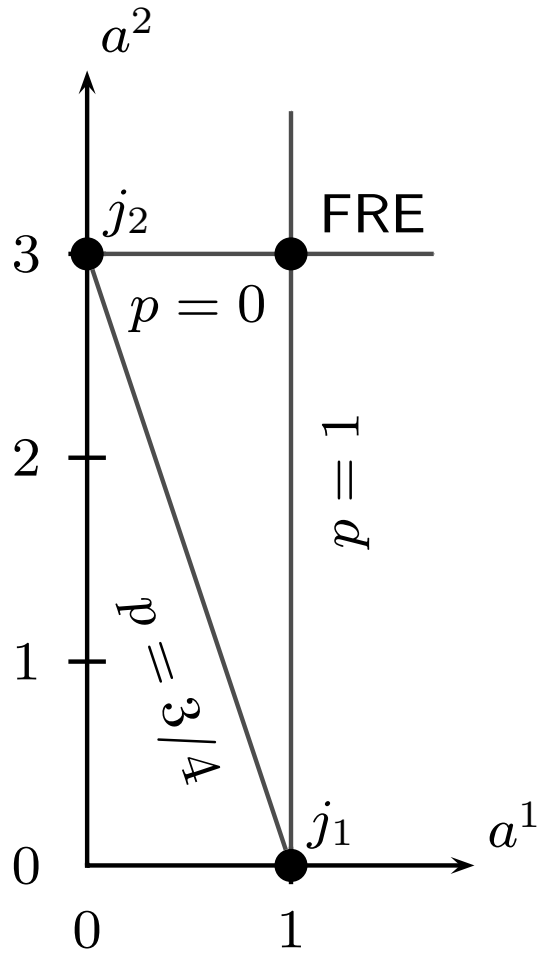


Figure 3: Modified equilibrium payoffs in Example 1

Full revelation of information (Example 2)

	j_1	j_2	
k_1	$1,1$	$0,0$	p
k_2	$0,0$	$3,3$	$(1-p)$

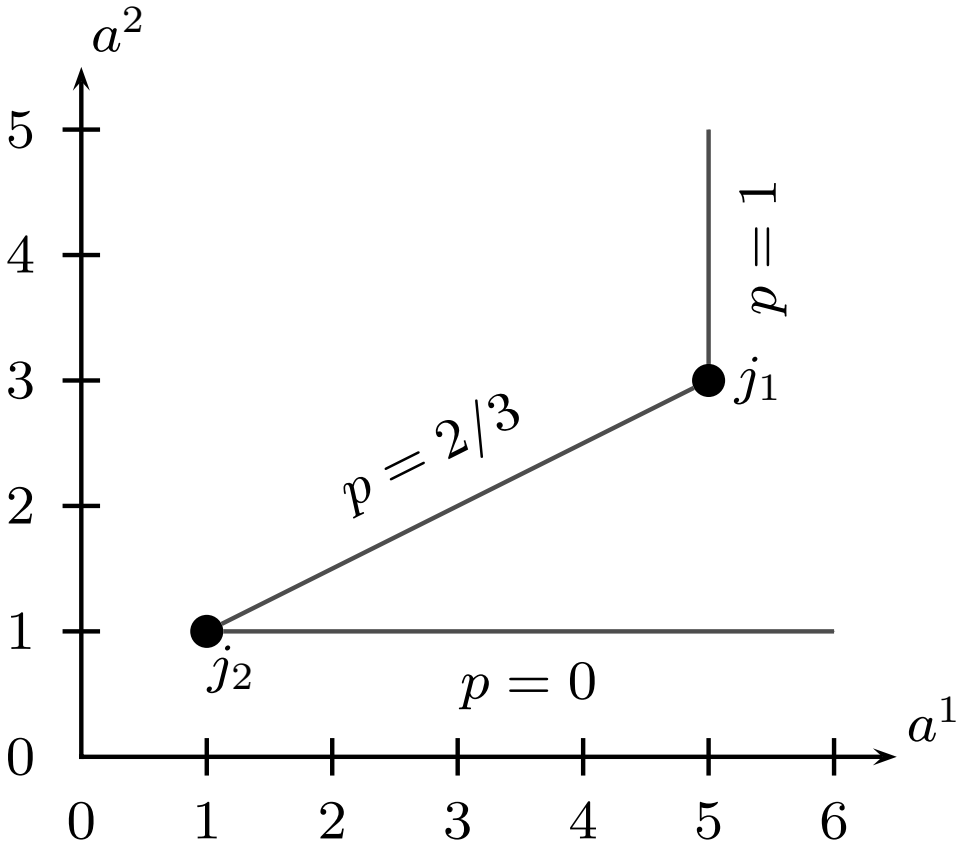
$$Y(p) = \begin{cases} \{j_1\} & \text{if } p > 3/4 \\ \{j_2\} & \text{if } p < 3/4 \\ \Delta(J) & \text{if } p = 3/4 \end{cases}$$



Unique equilibrium, non-revealing (Example 3)

	j_1	j_2	
k_1	$5,2$	$1,0$	p
k_2	$3,0$	$1,4$	$(1-p)$

$$Y(p) = \begin{cases} \{j_1\} & \text{if } p > 2/3, \\ \{j_2\} & \text{if } p < 2/3, \\ \Delta(J) & \text{if } p = 2/3 \end{cases}$$



Unique equilibrium, non-revealing (Example 4)

	j_1	j_2	
k_1	$3,2$	$4,0$	p
k_2	$3,0$	$1,4$	$(1-p)$

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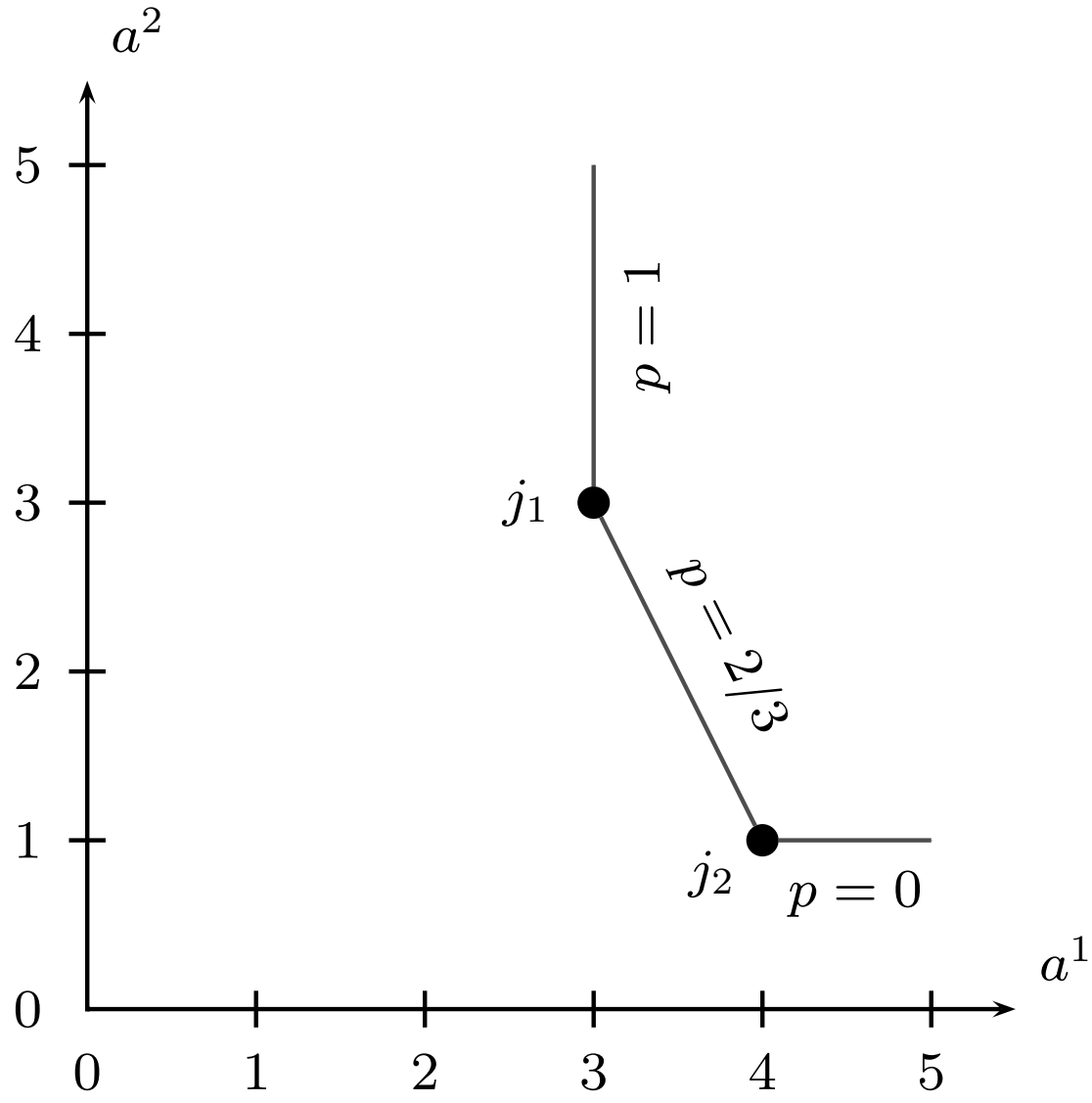
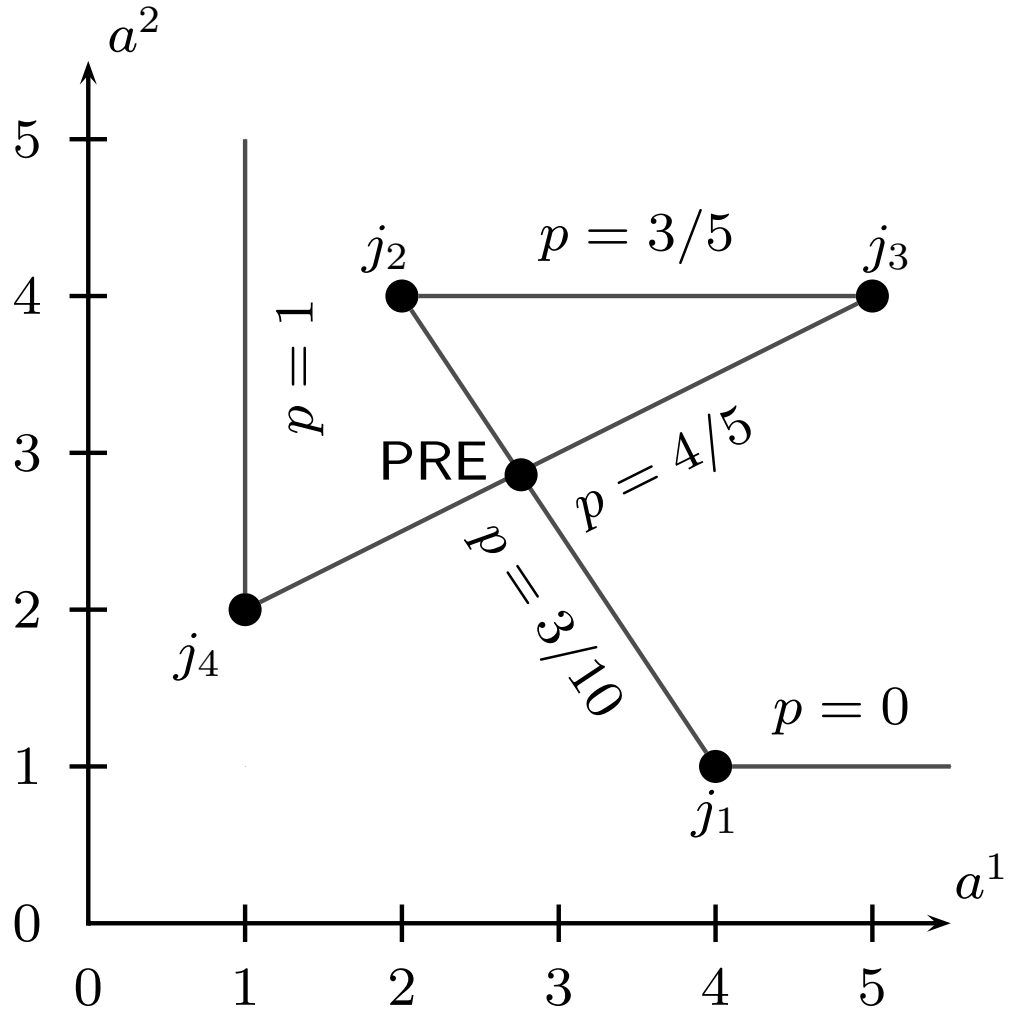


Partial revelation of information: Example 6

	j_1	j_2	j_3	j_4	
k_1	$4,0$	$2,7$	$5,9$	$1,10$	p
k_2	$1,10$	$4,7$	$4,4$	$2,0$	$1-p$

$$Y(p) = \begin{cases} \{j_1\} & \text{if } p < 3/10 \\ \Delta(\{j_1, j_2\}) & \text{if } p = 3/10 \\ \{j_2\} & \text{if } p \in (3/10, 3/5) \\ \Delta(\{j_2, j_3\}) & \text{if } p = 3/5 \\ \{j_3\} & \text{if } p \in (3/5, 4/5) \\ \Delta(\{j_3, j_4\}) & \text{if } p = 4/5 \\ \{j_4\} & \text{if } p > 4/5 \end{cases}$$



✍ Characterize explicitly players' strategies inducing the PRE when $p = 1/2$

Monotonic Games

Grossman (1981); Grossman and Hart (1980); Milgrom (1981); Milgrom and Roberts (1986); Watson (1996),. . .

$$A^k(j) > A^k(j') \;\Leftrightarrow\; j > j', \quad \forall\, k \in K$$

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Theorem (Monotonic games) *In a monotonic cheap talk games, every Nash equilibrium in which the decision maker uses pure strategies is non-revealing*

Proof. ✍ □

In particular, if $\arg\max_{j \in J} B^k(j)$ is unique for every k and depends on k, then there is no fully revealing equilibrium

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- If information is certifiable, then a fully revealing equilibrium always exists in monotonic games
- A FRE is also possible with public cheap talk to two decisionmakers, even if the private communication games are monotonic and have a unique non-revealing equilibrium

Example 7. The following monotonic game has a FRE:

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$$\sigma(k_1) = a \quad \sigma(k_2) = b$$

$$\tau(a) = \frac{2}{3}\, j_3 + \frac{1}{3}\, j_5 \quad \tau(b) = \frac{1}{6}\, j_2 + \frac{5}{6}\, j_4$$

	j_1	j_2	j_3	j_4	j_5
k_1	$1,2$	$2,0$	$3,3$	$4,0$	$5,3$
k_2	$1,2$	$2,3$	$3,0$	$4,3$	$5,0$

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Incomplete information on both sides: type $l \in L$ for the DM (private signal)

➥ Prior probability distribution $p \in \Delta(K \times L)$

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Example 9. The following monotonic game has a pure strategy FRE when $p = \begin{pmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{pmatrix}$:

$$\sigma(k_1) = \mathsf{a} \quad \sigma(k_2) = \mathsf{b},$$

$$\tau(\mathsf{a}, l_1) = \tau(\mathsf{b}, l_2) = j_2 \quad \tau(\mathsf{a}, l_2) = \tau(\mathsf{b}, l_1) = j_1.$$

	l_1		l_2	
	j_1	j_2	j_1	j_2
k_1	$1, 0$	$2, 2$	$1, 1$	$2, 0$
k_2	$1, 1$	$2, 0$	$1, 0$	$2, 2$

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Both players' preferences depend on the state: when t increases, both players want the action to increase but the ideal action of the expert, $a_1^*(t) = t + b$, is always higher than the ideal action of the decisionmaker, $a_2^*(t) = t$

Applications:

- Relationship between a doctor and his patient, where the patient has a bias towards excessive medication
- Choice of expenditure on a public project
- Choice of departure time for two friends (with different risk attitude) to take a plane (one having private information about flight time)
- Hierarchical relationships in organizations (e.g., choice = effort level)

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$$\sigma_1(t) = \begin{cases} m_1 & \text{if } t \in [0, x_1) \\ \vdots & \vdots \\ m_k & \text{if } t \in [x_{k-1}, x_k) \\ \vdots & \vdots \\ m_n & \text{if } t \in [x_{n-1}, 1] \end{cases}$$

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$$\Rightarrow \ \sigma_2(m_k) = \mathrm{E}(\boldsymbol{t} \mid m_k) = \mathrm{E}\left(\boldsymbol{t} \mid t \in [x_{k-1}, x_k)\right) = \frac{x_{k-1} + x_k}{2}$$

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Example: $m_1 = 0$, $m_2 = 1$ and $\mu(t \mid m) \sim \begin{cases} \mathcal{U}[0, x] & \text{if } m \in [0, x) \\ \mathcal{U}[x, 1] & \text{if } m \in [x, 1] \end{cases}$

Given the decisionmaker's strategy σ_2, the expert of type t will send the message $m \in \{m_1, m_2\}$ which induces the closest action to $t + b$

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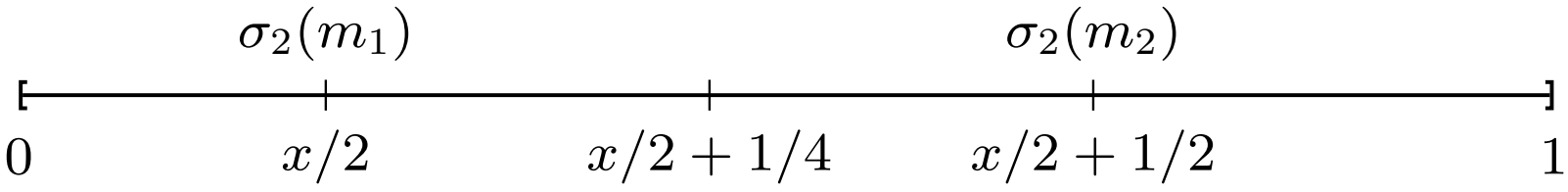

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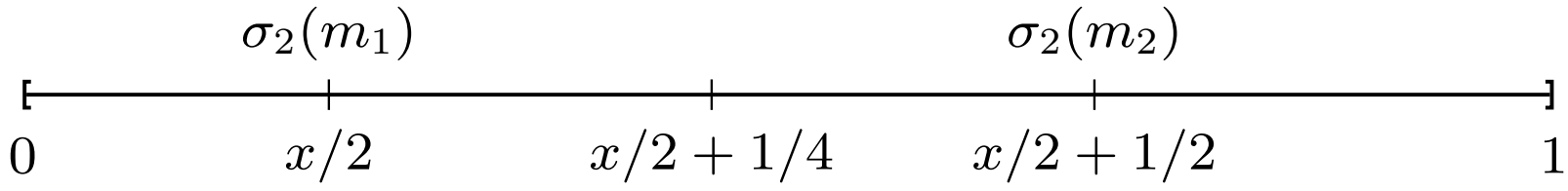

so $\sigma_1(t) = \begin{cases} m_1 & \text{if } \ t + b < x/2 + 1/4 \\ m_2 & \text{if } \ t + b > x/2 + 1/4 \end{cases}$

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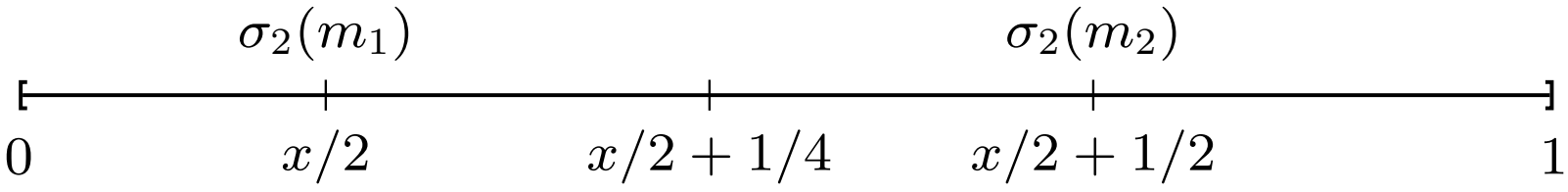

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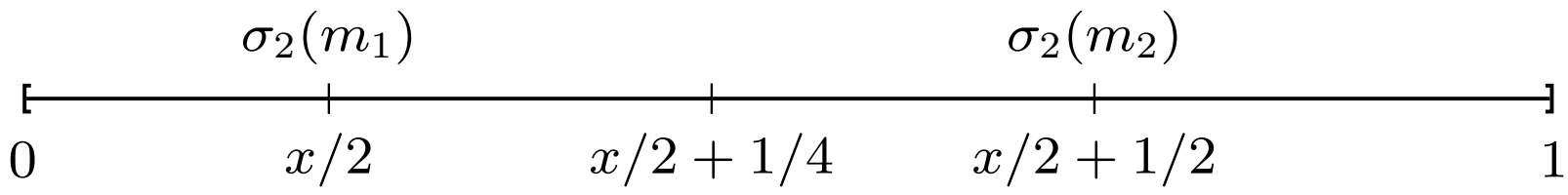

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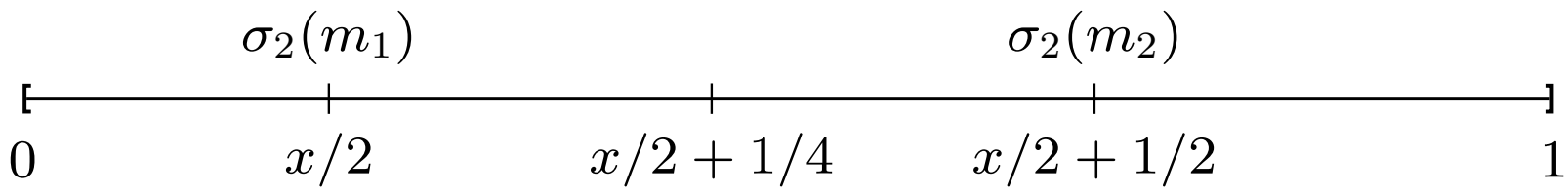
Given the decisionmaker's strategy σ_2, the expert of type t will send the message $m \in \{m_1, m_2\}$ which induces the closest action to $t + b$



so $\sigma_1(t) = \begin{cases} m_1 & \text{if } \; t + b < x/2 + 1/4 \\ m_2 & \text{if } \; t + b > x/2 + 1/4 \end{cases}$

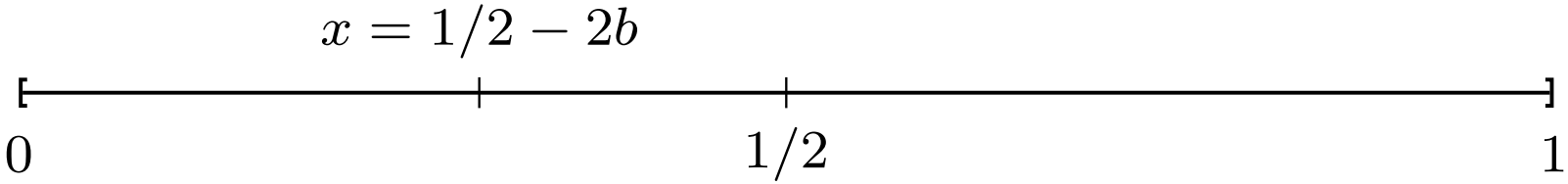
We started from

$$\sigma_1(t) = \begin{cases} m_1 & \text{if } \; t < x \\ m_2 & \text{if } \; t \geq x \end{cases} = \begin{cases} m_1 & \text{if } \; t + b < x + b \\ m_2 & \text{if } \; t + b \geq x + b \end{cases}$$

so we must have $x + b = x/2 + 1/4 \Leftrightarrow \boxed{x = 1/2 - 2\,b}$

- There is a 2-partitional equilibrium if and only if $b \leq 1/4$

➥ The interval $[x, 1]$ is $4\,b$ larger than $[0, x]$



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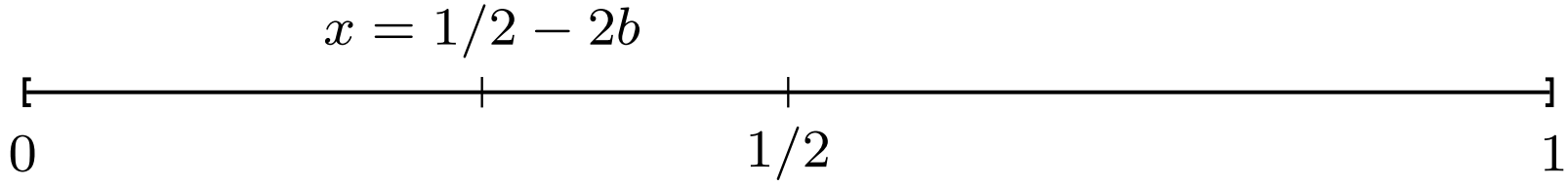

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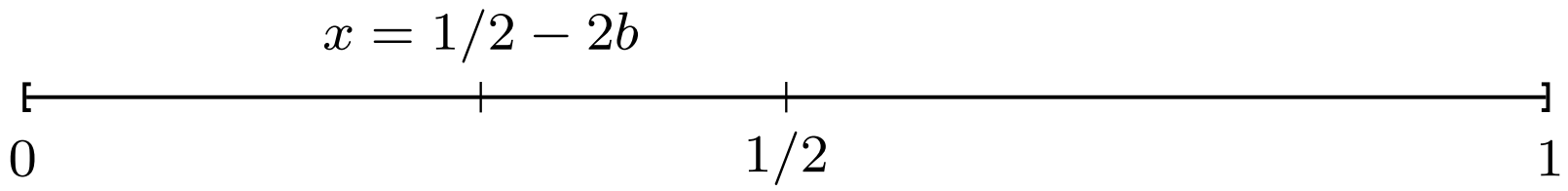

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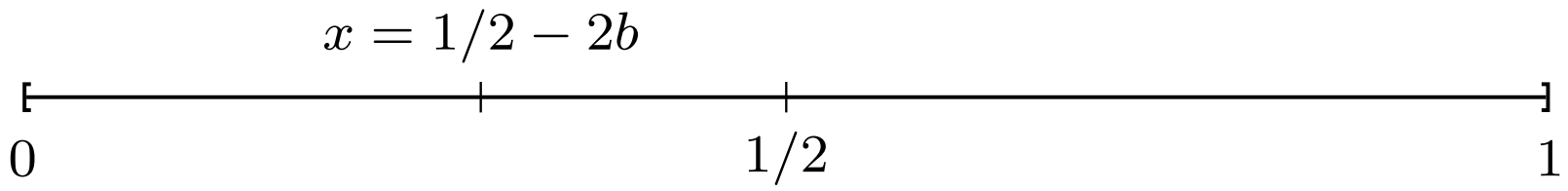

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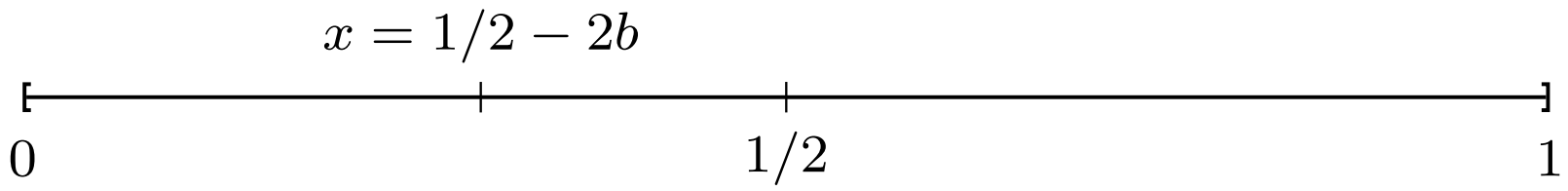

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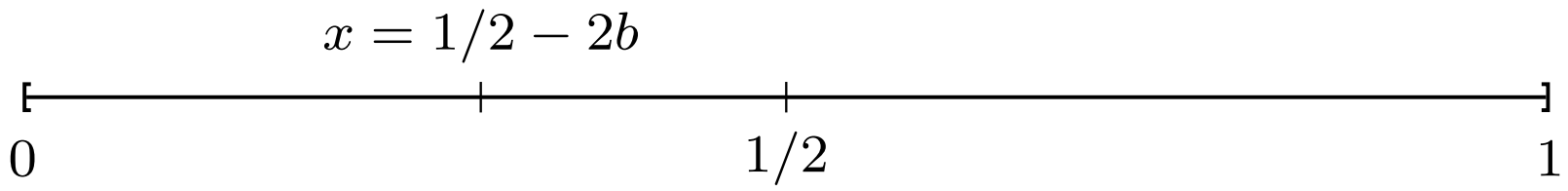

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so $[x_{k+1} - x_k] = [x_k - x_{k-1}] + 4\,b$

$$\Rightarrow \; x_k = x_1 + (x_1 + 4b) + (x_1 + 2\,(4b)) + \cdots + (x_1 + (k-1)\,(4b))$$

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but full revelation of information is impossible as long as players' preferences are not perfectly aligned ($b \neq 0$)

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- Characterize all equilibria when $b = 1/10$
- Verify that, in general, the best equilibrium for the expert depends on his type

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$$= 1/n + 2b\,(2k - n - 1)$$

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➠ With a fixed n, the expected payoff of the decisionmaker decreases with b

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so $EU_1 = EU_2 - b^2$ is also decreasing with b when n is fixed

Which equilibrium is the most efficient?

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Remark If information could be transmitted credibly, then the expected payoffs of both players would be higher than in all equilibria since we would have $EU_2 = 0$ and $EU_1 = -b^2$. We will see that the same outcome is achieved with certifiable information

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In general, equilibria cannot be compared in terms of efficiency anymore

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In this example, strategic money burning improves Pareto efficiency. The same phenomenon is possible in Crawford and Sobel's model (see Austen-Smith and Banks, 2000, 2002)

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Example: in a firm, instead of collecting all the information from the different hierarchical levels of the organization, a manager may delegate some decisions (e.g., investment decisions) to agents in lower levels of the hierarchies, even if these agents do not have exactly same incentives as the manager

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$$\begin{cases} EU_1 = EU_2 - b^2 \\ EU_2 = -\frac{1}{12n^2} - \frac{b^2(n^2-1)}{3} \end{cases}$$

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Of course, the expert always prefers delegation. The DM prefers delegation to cheap talk if $EU_2^D \geq EU_2 \Leftrightarrow b^2 \leq \frac{1}{12n^2} + \frac{b^2(n^2-1)}{3}$

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Dessein (2002) shows more generally (for a non-uniform prior distribution) that delegation is better than communication, except when the expert has a small informational advantage and communication is very noisy

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$$\max_{a(\cdot)} - \int_0^1 (a(t) - t)^2 \, dt$$
$$u.t.c. \; -(a(t) - t - b)^2 \geq -(a(t') - t - b)^2, \quad \forall \, t, t' \in T.$$

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Of course, if $b \neq 0$, the (first best) decision rule $a(t) = t$ does not satisfy the informational incentive constraint

The informational incentive constraint implies

$$a'(t)(a(t) - t - b) = 0, \quad \forall\, t \in T,$$

so on every interval $a(t)$ is either constant or $a(t) = t + b = a_1^*(t)$. In particular, full separation, $a(t) = t + b$, and full bunching, $a(t) = a$, satisfy the constraint

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Assuming continuity, the decision rule should take the following form, with $0 \leq t_1 \leq t_2 \leq 1$:

$$a(t) = \begin{cases} t_1 + b & \text{if } t \leq t_1, \\ t + b & \text{if } t \in [t_1, t_2], \\ t_2 + b & \text{if } t \geq t_2, \end{cases}$$

or should be constant on T

Hence, the principal minimizes

$$\int_0^{t_1} (\underbrace{a(t)}_{t_1+b} - t)^2 \, dt + \int_{t_1}^{t_2} (\underbrace{a(t)}_{t+b} - t)^2 \, dt + \int_{t_2}^{1} (\underbrace{a(t)}_{t_2+b} - t)^2 \, dt$$
$$= -(1/3)(b^3 - (t_1+b)^3) + b^2(t_2 - t_1) - (1/3)((t_2+b-1)^3 - b^3),$$

if $0 \leq t_1 \leq t_2 \leq 1$, or chooses $a(t) = 1/2$ for all t

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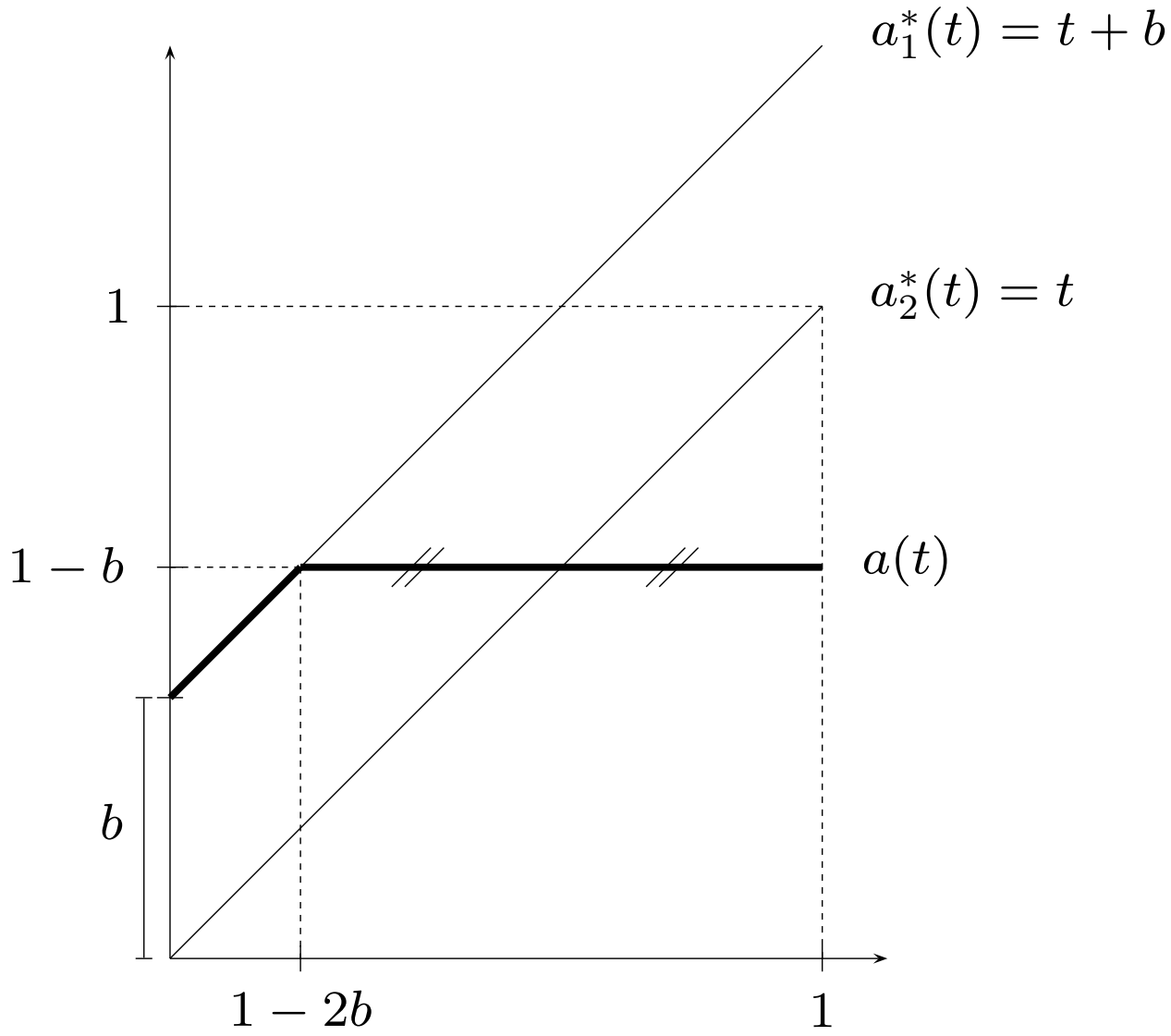

Comparing cheap talk, delegation (D) and commitment (C), we have:

$$EU_1^D \geq EU_1^C \geq EU_1,$$
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Remark. The optimal mechanism can be implemented with a delegation set $\mathcal{D} = [0, 1 - b]$, the principal letting the agent choose any action in $\mathcal{D}$

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Ideal points: $\theta + x_i \in \mathbb{R}^d$, where $x_p = 0$

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Fully Revealing Equilibrium (FRE):

$$\mu(\theta \mid s_1(\theta), s_2(\theta)) = 1, \quad \text{for all } \theta \in \Theta$$

Unidimensional Case.

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- Gilligan and Krehbiel (1989, American Journal of Political Science)
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- E.g., when $x_1, x_2 > 0$, there is a FRE $s_1(\theta) = s_2(\theta) = \theta$ with

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- When $x_1 < 0 < x_2$, a FRE exists if $|x_1| + |x_2|$ is not too large, but may rely on implausible (extreme) beliefs off the equilibrium path

Multidimensional Case.

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Proposition 1 (Battaglini, 2002) *If $d = 2$ and $x_1 \neq \alpha x_2$ for all $\alpha \in \mathbb{R}$ (i.e., x_1 and x_2 and linearly independent), then there is a FRE*

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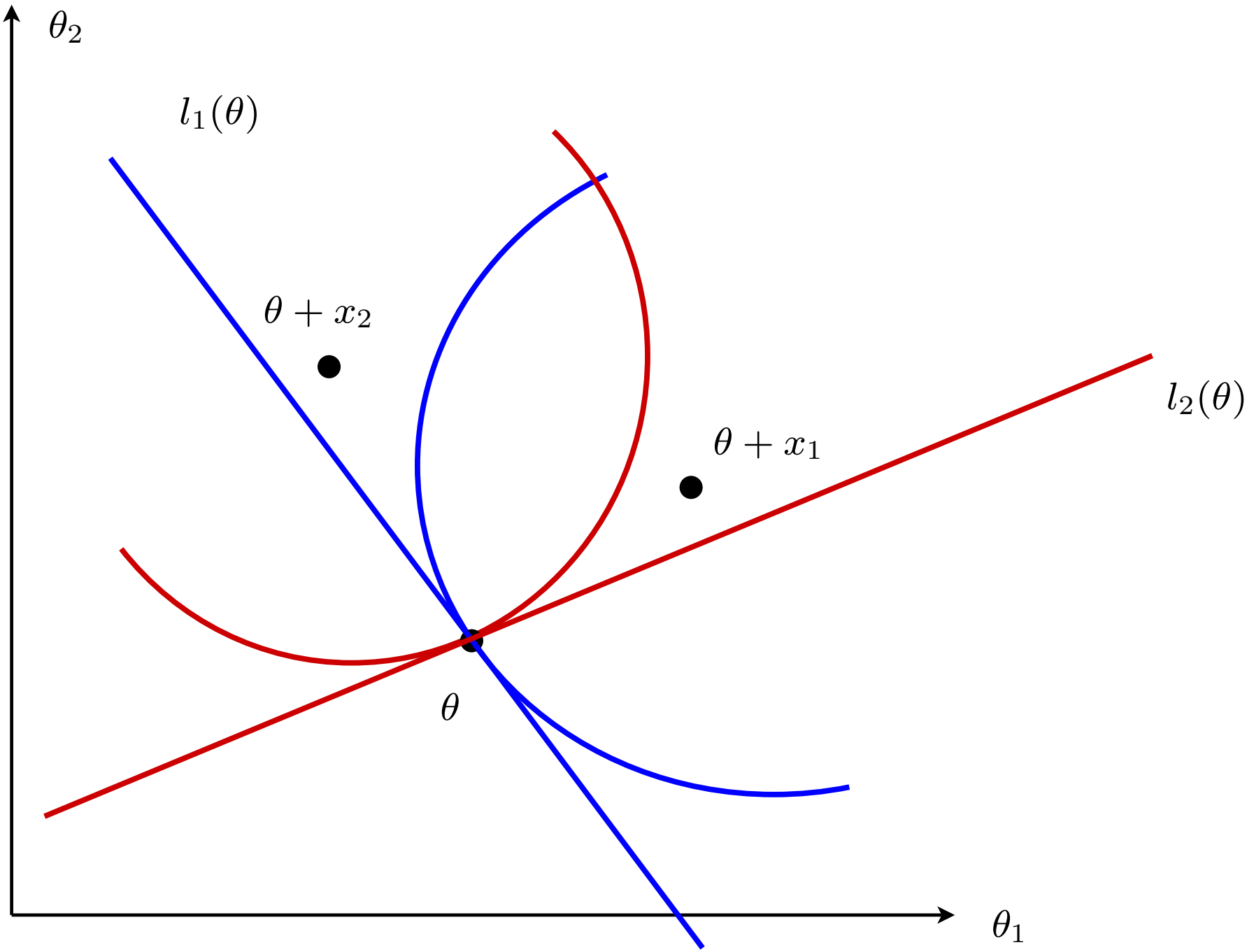
Proof.

Each expert will reveal the tangent of the other expert's indifference curve at the DM's ideal point θ


θ_2
$l_1(\theta)$
$\theta + x_2$
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θ
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The following strategy profile and beliefs constitute a FR PBE:

- $s_i(\theta) = l_j(\theta)$, $i \neq j$
- $\mu(s_1, s_2) = s_1 \cap s_2$ (and any point in $l_i(\theta)$ if $s_i \cap l_i(\theta) = \emptyset$)
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If expert i reveals $\hat{s}_i$ when the state is θ, then the action of the DM is

$$x(\hat{s}_i, s_j(\theta)) = \mu(\hat{s}_i, l_i(\theta)) \in l_i(\theta)$$

which, by construction, is the closest to i's ideal point when $\hat{s}_i = l_j(\theta)$ $\square$

Remark The result can be extended to more than two dimensions of the policy space and to quasi-concave utilities (not necessarily quadratic), but may not be robust to the timing of the game (sequential cheap talk)

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Example:

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	q_1	q_2	r_1	r_2
k_1	v_1, x_1	$0, 0$	w_1, y_1	$0, 0$
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Mutual discipline: There is no separating equilibrium in private, but there is in public. E.g., when $v_1 = w_2 = 3$ and $v_2 = w_1 = -1$

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- 12 repetitions among 8 subjects with random matching

Results:

- Observed average distance between states and actions increases with the bias b
- Receivers' average payoffs decrease with b
- Two much information is revealed when it should not (b_4, b_5)

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Results:

- Observed correlation between
  - states and actions
  - messages and actions
  - states and messages

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The unique communication equilibrium is non-revealing (monotonic game)

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Results:

- Without communication possibility, actual efficiency close to theoretical efficiency
- With cheap talk communication, the adverse selection problem is not as severe as predicted
  - efficiency is significantly higher than predicted
  - but at the expense of buyers (they overpay by relying on sellers' exaggerated claims)
- With certifiable information,
  - efficiency is smaller than predicted, but higher than under cheap talk
  - no wealth transfer from buyers to sellers anymore

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