Strategic Information Transmission: Cheap Talk Games
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Outline
(November 12, 2008)

- Credible information under cheap talk: Examples
- Geometric characterization of Nash equilibrium outcomes
- Expertise with a biased interested party
- Communication in organizations: Delegation vs. cheap talk vs. commitment
- Multiple Senders and Multidimensional Cheap Talk
- Lobbying with several audiences
- Some experimental evidence
General References:


- Myerson (1991, chap. 6): “Games of communication,” in “Game Theory, Analysis of Conflict”

- Sobel (2007): “Signalling Games”
**Cheap talk** = communication which is
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- strategic and non-binding (no contract, no commitment)
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In its simplest form, a cheap talk game in a specific signaling games in which messages are costless (i.e., do not enter into players' utility functions)
Example 1. (Signal of productivity in a labor market)

Extremely simplified version of Spence (1973) model of education:
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affect his productivity, but is **costly**
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Perfect competition among employers, so the employer chooses a salary equal to the expected productivity of the worker (zero expected profits)

The worker chooses a level of education $e \in \{e_L, e_H\} = \{0, 3\}$ (which does not affect his productivity, but is costly)

$$
\begin{align*}
A^k(j) &= j - c(k, e) = j - e/k \quad \text{(worker)} \\
B^k(j) &= -[k - j]^2 \quad \text{(employer)}
\end{align*}
$$
Figure 1: Fully revealing equilibrium in the labor market signaling game (example 1)
What happens if we replace the level of education $e$ by cheap talk?
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Then, the message “my ability is high” is not credible anymore: whatever his type, the worker always wants the employer to believe that his ability is high (in order to get a high salary)
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<table>
<thead>
<tr>
<th>$j_H$</th>
<th>$j_M$</th>
<th>$j_L$</th>
<th>$k_L$</th>
<th>$k_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, -4</td>
<td>2, -1</td>
<td>1, 0</td>
<td>3, 0</td>
<td>2, -1</td>
</tr>
</tbody>
</table>

$\Pr(k_L) = 1/2$

$\Pr(k_H) = 1/2$
Associated one-shot cheap talk game with two possible messages
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(3, −4)  (2, −1)  (1, 0)  (3, 0)  (2, −1)  (1, −4)
Associated one-shot cheap talk game with two possible messages

\[
\begin{pmatrix}
(3, -4) & (2, -1) & (1, 0) & (3, 0) & (2, -1) & (1, -4)
\end{pmatrix}
\]

Employer

Worker

Employer

Worker

Fully revealing equilibrium?
Associated one-shot cheap talk game with two possible messages

(3, -4)  (2, -1)  (1, 0)  (3, 0)  (2, -1)  (1, -4)

Worker

Employer

Worker

Employer

(3, -4)  (2, -1)  (1, 0)  (3, 0)  (2, -1)  (1, -4)

Fully revealing equilibrium? No, because the worker of type $k_L$ deviates by sending the same message as the worker of type $k_H$. 
Associated one-shot cheap talk game with two possible messages

(3, −4)  (2, −1)  (1, 0)  (3, 0)  (2, −1)  (1, −4)

Fully revealing equilibrium?  No, because the worker of type $k_L$ deviates by sending the same message as the worker of type $k_H$

✉️ Non-revealing equilibrium?  Yes, a NRE always exists in cheap talk games
Can cheap talk be credible and help to transmit relevant information?
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Example 2. (Credible information revelation)
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\[
\begin{array}{c|cc}
  & j_1 & j_2 \\
  \hline
  k_1 & 1, 1 & 0, 0 \\
  \hline
  k_2 & 0, 0 & 3, 3 \\
  \hline
\end{array}
\]

\[ p, (1 - p) \]
Can cheap talk be credible and help to transmit relevant information?

**Example 2. (Credible information revelation)**

<table>
<thead>
<tr>
<th></th>
<th>$j_1$</th>
<th>$j_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>1, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>$k_2$</td>
<td>0, 0</td>
<td>3, 3</td>
</tr>
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</table>

$$Y(p) = \begin{cases} 
\{j_1\} & \text{if } p > \frac{3}{4}, \\
\{j_2\} & \text{if } p < \frac{3}{4}, \\
\Delta(J) & \text{if } p = \frac{3}{4}.
\end{cases}$$
Can cheap talk be credible and help to transmit relevant information?

Example 2. (Credible information revelation)

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\end{cases}$

The sender’s preference over the receiver’s beliefs are positively correlated with the truth.
Figure 2: Fully revealing equilibrium in Example 2.
Example 3. (Revelation of information which is not credible)
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<table>
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<th>$j_1$</th>
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</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>5,2</td>
<td>1,0</td>
</tr>
<tr>
<td>$k_2$</td>
<td>3,0</td>
<td>1,4</td>
</tr>
</tbody>
</table>

$p$, $(1 - p)$
Example 3. (Revelation of information which is not credible)

\[
\begin{array}{ccc}
  j_1 & j_2 & p \\
- & - & - \\
  \ \ k_1 & 5,2 & 1,0 \\
  \ \ k_2 & 3,0 & 1,4 \\
  \ \ & (1-p) & \ \\
\end{array}
\]

\[
Y(p) = \begin{cases} 
  \{j_1\} & \text{if } p > 2/3, \\
  \{j_2\} & \text{if } p < 2/3, \\
  \Delta(J) & \text{if } p = 2/3. 
\end{cases}
\]
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\[
Y(p) = \begin{cases} 
  \{j_1\} & \text{if } p > \frac{2}{3}, \\
  \{j_2\} & \text{if } p < \frac{2}{3}, \\
  \Delta(J) & \text{if } p = \frac{2}{3}.
\end{cases}
\]

The sender’s preference over the receiver’s beliefs is **not correlated** with the truth. The unique equilibrium of the cheap talk game in NR, even if when \( p < \frac{2}{3} \) communication of information would increase both players’ payoffs.
Example 4. (Revelation of information which is not credible)
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<tbody>
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<td>4, 0</td>
</tr>
<tr>
<td>$p$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_2$</td>
<td>3, 0</td>
<td>1, 4</td>
</tr>
<tr>
<td>$(1 - p)$</td>
<td></td>
<td></td>
</tr>
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</table>
Example 4. (Revelation of information which is not credible)

\[
\begin{array}{ccc}
  j_1 & j_2 \\
  k_1 & 3, 2 & 4, 0 \\
  k_2 & 3, 0 & 1, 4 \\
\end{array}
\]

\[ Y(p) = \begin{cases} 
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The sender’s preference over the receiver’s beliefs is **negatively correlated** with the truth. The unique equilibrium of the cheap talk game in NR
Example 5. (Partial revelation of information)
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<tr>
<th></th>
<th>(j_1)</th>
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<th>(j_3)</th>
<th>(j_4)</th>
<th>(j_5)</th>
<th>(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_1)</td>
<td>1, 10</td>
<td>3, 8</td>
<td>0, 5</td>
<td>3, 0</td>
<td>1, −8</td>
<td></td>
</tr>
<tr>
<td>(k_2)</td>
<td>1, −8</td>
<td>3, 0</td>
<td>0, 5</td>
<td>3, 8</td>
<td>1, 10</td>
<td>1 − (p)</td>
</tr>
</tbody>
</table>

\[ Y(p) = \begin{cases} 
\{j_5\} & \text{if } p < 1/5 \\
\{j_4\} & \text{if } p \in (1/5, 3/8) \\
\{j_3\} & \text{if } p \in (3/8, 5/8) \\
\{j_2\} & \text{if } p \in (5/8, 4/5) \\
\{j_1\} & \text{if } p > 4/5 
\end{cases} \]
Partially revealing equilibrium when $p = 1/2$:

\[
\begin{align*}
\sigma(k_1) &= \frac{3}{4}a + \frac{1}{4}b \\
\sigma(k_2) &= \frac{1}{4}a + \frac{3}{4}b
\end{align*}
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\end{align*}$$

$$\begin{align*}
\Pr(k_1 \mid a) &= \frac{\Pr(a \mid k_1) \Pr(k_1)}{\Pr(a)} = \frac{3}{4} \\
\Pr(k_1 \mid b) &= \frac{\Pr(b \mid k_1) \Pr(k_1)}{\Pr(b)} = \frac{1}{4}
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\[
\begin{align*}
\tau(a) &= j_2 \\
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\]

\[
\begin{align*}
\tau(a) &= j_2 \\
\tau(b) &= j_4
\end{align*}
\]

\[
\Rightarrow \text{ equilibrium, expected utility } = \frac{3}{4}(3, 8) + \frac{1}{4}(3, 0) = (3, 6) \text{ (better for the sender than the NRE and FRE)}
\]
Basic Decision Problem
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Two players
Two players

Player 1 = sender, expert (with no decision)
Two players

Player 1 = sender, expert (with no decision)

Player 2 = receiver, decisionmaker (with no information)
Two players

Player 1 = sender, expert (with no decision)
Player 2 = receiver, decisionmaker (with no information)

Two possible types for the expert (can be easily generalized):
\[ K = \{k_1, k_2\} = \{1, 2\}, \Pr(k_1) = p, \Pr(k_2) = 1 - p \]
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Action of the decisionmaker: \( j \in J \)
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$K = \{k_1, k_2\} = \{1, 2\}$, $\Pr(k_1) = p$, $\Pr(k_2) = 1 - p$

Action of the decisionmaker: $j \in J$

Payoffs: $A^k(j)$ and $B^k(j)$
Silent Game

\[ \Gamma(p) \]

\[ \begin{array}{cccc}
1 & \cdots & j & \cdots \\
A^1(1), B^1(1) & \cdots & A^1(j), B^1(j) & \cdots \\
\end{array} \]

\[ \begin{array}{cccc}
1 & \cdots & j & \cdots \\
A^2(1), B^2(1) & \cdots & A^2(j), B^2(j) & \cdots \\
\end{array} \]
• Mixed action of the DM: \( y \in \Delta(J) \)
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\[ A^k(y) = \sum_{j \in J} y(j) A^k(j) \]

\[ B^k(y) = \sum_{j \in J} y(j) B^k(j) \]
• Mixed action of the DM: \( y \in \Delta(J) \)

\[ \Rightarrow \text{expected payoffs} \begin{cases} 
  A^k(y) = \sum_{j \in J} y(j) A^k(j) \\
  B^k(y) = \sum_{j \in J} y(j) B^k(j) 
\end{cases} \]

• Optimal mixed actions in \( \Gamma(p) \) (non-revealing “equilibria”):
• Mixed action of the DM: \( y \in \Delta(J) \)

\[
A^k(y) = \sum_{j \in J} y(j) A^k(j)
\]

\[
B^k(y) = \sum_{j \in J} y(j) B^k(j)
\]

\Rightarrow \text{expected payoffs}

• Optimal mixed actions in \( \Gamma(p) \) (non-revealing “equilibria”):

\[
Y(p) \equiv \arg \max_{y \in \Delta(J)} p B^1(y) + (1 - p) B^2(y)
\]

\[
= \{ y : p B^1(y) + (1 - p) B^2(y) \geq p B^1(j) + (1 - p) B^2(j), \ \forall \ j \in J \} 
\]
• Mixed action of the DM: \( y \in \Delta(J) \)

\[
\begin{align*}
A^k(y) &= \sum_{j \in J} y(j) A^k(j) \\
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\]

⇒ expected payoffs

• Optimal mixed actions in \( \Gamma(p) \) (non-revealing “equilibria”):

\[
Y(p) \equiv \arg \max_{y \in \Delta(J)} p B^1(y) + (1 - p) B^2(y)
\]

\[
= \{ y : p B^1(y) + (1 - p) B^2(y) \geq p B^1(j) + (1 - p) B^2(j), \ \forall \ j \in J \}
\]

**Remark**  Mixed actions are used in the communication extension of the game to construct equilibria in which the expert is indifferent between several messages. They also serve as punishments off the equilibrium path in communication games with certifiable information (persuasion games)
• “Equilibrium” payoffs in $\Gamma(p)$:
• “Equilibrium” payoffs in $\Gamma(p)$:

$$\mathcal{E}(p) \equiv \{(a, \beta) : \exists y \in Y(p), \ a = A(y), \ \beta = p B^1(y) + (1 - p) B^2(y)\}$$
Unilateral information transmission from the expert to the decisionmaker
Unilateral information transmission from the expert to the decisionmaker

Set of messages ("keyboard") of the expert:

\[ M = \{a, b, \ldots, \}, \quad 3 \leq |M| < \infty \]
Unilateral Communication Game $\Gamma^0_S(p)$

Unilateral information transmission from the expert to the decisionmaker

Set of messages (“keyboard”) of the expert:

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<table>
<thead>
<tr>
<th>Information Phase</th>
<th>Communication phase</th>
<th>Action phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>The expert learns $k \in K$</td>
<td>The expert sends $m \in M$</td>
<td>The DM chooses $j \in J$</td>
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Unilateral Communication Game $\Gamma^0_S(p)$

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Information Phase
The expert learns $k \in K$

Communication phase
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Action phase
The DM chooses $j \in J$

Strategy of the expert: $\sigma : K \to \Delta(M)$
Unilateral Communication Game $\Gamma^0_S(p)$

Unilateral information transmission from the expert to the decisionmaker

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$$M = \{a, b, \ldots, \}, \quad 3 \leq |M| < \infty$$

Information Phase

The expert learns $k \in K$

Communication phase

The expert sends $m \in M$

Action phase

The DM chooses $j \in J$

Strategy of the expert: $\sigma : K \rightarrow \Delta(M)$

Strategy of the DM: $\tau : M \rightarrow \Delta(J)$
Example. Two messages \((M = \{a, b\})\)
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Example. Two messages \( M = \{a, b\} \)

\[ \ldots (A^1(j), B^1(j)) \ldots \]

\[ \ldots (A^2(j), B^2(j)) \ldots \]

\[ \mathcal{E}_S^0(p): \text{Equilibrium payoffs of } \Gamma_S^0(p) \]
Characterization of NE Payoffs of $\Gamma^0_S(p)$

Recall. $\mathcal{E}(p) \subseteq \mathbb{R}^2 \times \mathbb{R}$: NE payoffs in the silent game $\Gamma(p)$
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**Modified equilibrium payoffs** of $\Gamma(p)$: $\mathcal{E}^+(p)$: the expert can have a (virtual) payoff which is higher than his equilibrium payoff when his type has zero probability.
Recall. $\mathcal{E}(p) \subseteq \mathbb{R}^2 \times \mathbb{R}$: NE payoffs in the silent game $\Gamma(p)$

**Modified equilibrium payoffs** of $\Gamma(p)$: $\mathcal{E}^+(p)$: the expert can have a (virtual) payoff which is higher than his equilibrium payoff when his type has zero probability

$$(a, \beta) \in \mathbb{R}^2 \times \mathbb{R}$$
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Graph of the modified equilibrium payoff correspondence:
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Graph of the modified equilibrium payoff correspondence:

$$\text{gr } E^+ \equiv \{(a, \beta, p) \in \mathbb{R}^2 \times \mathbb{R} \times [0, 1] : (a, \beta) \in E^+(p)\}$$
Hart (1985, MOR), Aumann and Hart (2003, Ecta): Without any assumption on the utility functions, all equilibrium payoffs of the unilateral communication game $\Gamma^0_S(p)$ can be geometrically characterized only from the graph of the equilibrium payoff correspondence of the silent game.
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**Theorem (Characterization of $\mathcal{E}_S^0(p)$)** Let $p \in (0, 1)$. A payoff profile $(a, \beta)$ is a Nash equilibrium payoff of the unilateral communication game $\Gamma^0_S(p)$ if and only if $(a, \beta, p)$ belongs to $\text{conv}_a(\text{gr} \mathcal{E}^+)$, the set points obtained by convexification of the set $\text{gr} \mathcal{E}^+$ in $(\beta, p)$ by keeping the expert's payoff, $a$, constant:

$$\mathcal{E}_S^0(p) = \{(a, \beta) \in \mathbb{R}^2 \times \mathbb{R} : (a, \beta, p) \in \text{conv}_a(\text{gr} \mathcal{E}^+)\}$$
Illustrations
Unique equilibrium, non revealing (Example 1)
Unique equilibrium, non revealing (Example 1)

Optimal decisions in the silent game:

\[ Y(p) = \begin{cases} 
\{j_H\} & \text{if } p < 1/4 \\
\Delta(\{j_H, j_M\}) & \text{if } p = 1/4 \\
\{j_M\} & \text{if } p \in (1/4, 3/4) \\
\Delta(\{j_M, j_L\}) & \text{if } p = 3/4 \\
\{j_L\} & \text{if } p > 3/4 
\end{cases} \]
Figure 3: Modified equilibrium payoffs in Example 1
Full revelation of information (Example 2)

\[
\begin{array}{cc}
  j_1 & j_2 \\
  k_1 & 1,1 & 0,0 & p \\
  k_2 & 0,0 & 3,3 & (1 - p)
\end{array}
\]

\[
Y(p) = \begin{cases} 
\{j_1\} & \text{if } p > \frac{3}{4} \\
\{j_2\} & \text{if } p < \frac{3}{4} \\
\Delta(J) & \text{if } p = \frac{3}{4}
\end{cases}
\]
Unique equilibrium, non-revealing (Example 3)

\[
Y(p) = \begin{cases} 
\{j_1\} & \text{if } p > \frac{2}{3}, \\
\{j_2\} & \text{if } p < \frac{2}{3}, \\
\Delta(J) & \text{if } p = \frac{2}{3}
\end{cases}
\]
Unique equilibrium, non-revealing (Example 4)

\[ Y(p) = \begin{cases} 
\{j_1\} & \text{if } p > \frac{2}{3}, \\
\{j_2\} & \text{if } p < \frac{2}{3}, \\
\Delta(J) & \text{if } p = \frac{2}{3} 
\end{cases} \]
Partial revelation of information: Example 6

\[ Y(p) = \begin{cases} 
\{j_1\} & \text{if } p < 3/10 \\
\Delta(\{j_1, j_2\}) & \text{if } p = 3/10 \\
\{j_2\} & \text{if } p \in (3/10, 3/5) \\
\Delta(\{j_2, j_3\}) & \text{if } p = 3/5 \\
\{j_3\} & \text{if } p \in (3/5, 4/5) \\
\Delta(\{j_3, j_4\}) & \text{if } p = 4/5 \\
\{j_4\} & \text{if } p > 4/5 
\end{cases} \]
Characterize explicitly players’ strategies inducing the PRE when $p = 1/2$
Grossman (1981); Grossman and Hart (1980); Milgrom (1981); Milgrom and Roberts (1986); Watson (1996), . . .

\[ A^k(j) > A^k(j') \iff j > j', \quad \forall k \in K \]
Monotonic Games

Grossman (1981); Grossman and Hart (1980); Milgrom (1981); Milgrom and Roberts (1986); Watson (1996), ...  

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Examples:

- A seller who wants to maximize sells
- A manager who wants to maximize the value of the firm
- A worker who wants the job with the highest wage (whatever his competence)
- A firm who wants its competitors to decrease their productions
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- A seller who wants to maximize sells
- A manager who wants to maximize the value of the firm
- A worker who wants the job with the highest wage (whatever his competence)
- A firm who wants its competitors to decrease their productions

**Theorem (Monotonic games)** In a monotonic cheap talk games, every Nash equilibrium in which the decision maker uses pure strategies is non-revealing

Proof. \(\square\)
In particular, if \( \arg\max_{j \in J} B^k(j) \) is unique for every \( k \) and depends on \( k \), then there is no fully revealing equilibrium.
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But information transmission is still possible in monotonic games:

- A fully revealing equilibrium may exist if the DM uses mixed strategies (Example 7)
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But information transmission is still possible in monotonic games

- A fully revealing equilibrium may exist if the DM uses mixed strategies (Example 7)
- Even if \( \arg \max_{j \in J} B^k(j) \) is unique for every \( k \), a partially revealing equilibrium may exist (Example 8)
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- If the DM also has private information (incomplete information on both sides), a fully revealing equilibrium in pure strategy may exist.
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- If information is certifiable, then a fully revealing equilibrium always exists in monotonic games
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- If information is certifiable, then a fully revealing equilibrium always exists in monotonic games.

- A FRE is also possible with public cheap talk to two decisionmakers, even if the private communication games are monotonic and have a unique non-revealing equilibrium.
Example 7. The following monotonic game has a FRE:
Example 7. The following monotonic game has a FRE:

\[ \sigma(k_1) = a \quad \sigma(k_2) = b \]

\[ \tau(a) = \frac{2}{3} j_3 + \frac{1}{3} j_5 \quad \tau(b) = \frac{1}{6} j_2 + \frac{5}{6} j_4 \]

<table>
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<th>( k_1 )</th>
<th>( j_1 )</th>
<th>( j_2 )</th>
<th>( j_3 )</th>
<th>( j_4 )</th>
<th>( j_5 )</th>
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Example 8. The following monotonic game has a PRE when $\Pr[k_1] = 3/10$: 
Example 8. The following monotonic game has a PRE when \( \Pr[k_1] = 3/10 \):

\[
\sigma(k_1) = \frac{1}{3} a + \frac{2}{3} b \quad \sigma(k_2) = \frac{4}{7} a + \frac{3}{7} b
\]

\[
\tau(a) = \frac{1}{3} j_1 + \frac{2}{3} j_3 \quad \tau(b) = \frac{2}{3} j_2 + \frac{1}{3} j_3
\]

<table>
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<th></th>
<th>( j_1 )</th>
<th>( j_2 )</th>
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<td>2, 0</td>
<td>3, 4</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>1, 7</td>
<td>2, 10</td>
<td>3, 9</td>
</tr>
</tbody>
</table>
Incomplete information on both sides: type $l \in L$ for the DM (private signal)

$$\Rightarrow \text{ Prior probability distribution } p \in \Delta(K \times L)$$
Incomplete information on both sides: type $l \in L$ for the DM (private signal)

Prior probability distribution $p \in \Delta(K \times L)$

Example 9. The following monotonic game has a pure strategy FRE when

$p = \begin{pmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{pmatrix}$:

$\sigma(k_1) = a \quad \sigma(k_2) = b,$

$\tau(a, l_1) = \tau(b, l_2) = j_2 \quad \tau(a, l_2) = \tau(b, l_1) = j_1.$
Crawford and Sobel’s (1982) Model
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- Types of the expert: $T = [0, 1]$, uniformly distributed
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- Types of the expert: $T = [0, 1]$, uniformly distributed
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- Utility of the decisionmaker (player 2): $u_2(a; t) = -[a - t]^2$

Both players’ preferences depend on the state: when $t$ increases, both players want the action to increase but the ideal action of the expert, $a_1^*(t) = t + b$, is always higher than the ideal action of the decisionmaker, $a_2^*(t) = t$
Applications:

- Relationship between a doctor and his patient, where the patient has a bias towards excessive medication
- Choice of expenditure on a public project
- Choice of departure time for two friends (with different risk attitude) to take a plane (one having private information about flight time)
- Hierarchical relationships in organizations (e.g., choice = effort level)
“$n$-partitional” equilibria, in which $n$ different messages are sent:
“n-partitional” equilibria, in which \( n \) different messages are sent:

\[
\sigma_1(t) = \begin{cases} 
    m_1 & \text{if } t \in [0, x_1) \\
    \vdots & \vdots \\
    m_k & \text{if } t \in [x_{k-1}, x_k) \\
    \vdots & \vdots \\
    m_n & \text{if } t \in [x_{n-1}, 1]
\end{cases}
\]

where \( 0 < x_1 < \cdots < x_{n-1} < x_n = 1 \) and \( m_k \neq m_l \ \forall \ k \neq l \)
“n-partitional” equilibria, in which \( n \) different messages are sent:

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and \( n \leq n^*(b) = \text{maximal number of different messages that can be sent in equilibrium, decreasing with } b \)
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$$
\Rightarrow \sigma_2(m_k) = E(t \mid m_k) = E(t \mid t \in [x_{k-1}, x_k]) = \frac{x_{k-1} + x_k}{2}
$$
Equilibrium conditions for $n = 2$
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\sigma_1(t) = \begin{cases} 
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  m_2 & \text{if } t \in [x, 1]
\end{cases}
\]
Equilibrium conditions for $n = 2$

$$\sigma_1(t) = \begin{cases} m_1 & \text{if } t \in [0, x) \\ m_2 & \text{if } t \in [x, 1] \end{cases} \Rightarrow \sigma_2(m) = \begin{cases} x/2 & \text{if } m = m_1 \\ (x + 1)/2 & \text{if } m = m_2 \end{cases}$$
Equilibrium conditions for $n = 2$

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For off the equilibrium path messages $m \notin \{m_1, m_2\}$, it suffices to consider the same beliefs as along the equilibrium path.
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For off the equilibrium path messages $m \notin \{m_1, m_2\}$, it suffices to consider the same beliefs as along the equilibrium path.

Example: $m_1 = 0$, $m_2 = 1$ and $\mu(t \mid m) \sim \begin{cases} \mathcal{U}[0, x] & \text{if } m \in [0, x) \\ \mathcal{U}[x, 1] & \text{if } m \in [x, 1] \end{cases}$
Given the decisionmaker’s strategy $\sigma_2$, the expert of type $t$ will send the message $m \in \{m_1, m_2\}$ which induces the closest action to $t + b$. 
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\[
\begin{array}{cccc}
\sigma_2(m_1) & & \sigma_2(m_2) \\
0 & x/2 & x/2 + 1/4 & x/2 + 1/2 & 1
\end{array}
\]

so $\sigma_1(t) = \begin{cases} 
m_1 & \text{if } t + b < x/2 + 1/4 \\
m_2 & \text{if } t + b > x/2 + 1/4 \end{cases}$
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$$
\begin{array}{c|c|c|c|}
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We started from

$$
\sigma_1(t) = \begin{cases} 
m_1 & \text{if } t < x \\
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$$

so we must have $x + b = x/2 + 1/4 \iff x = 1/2 - 2b$
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\begin{array}{c|c|c|c}
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\end{cases} = \begin{cases} 
  m_1 & \text{if } t + b < x + b \\
  m_2 & \text{if } t + b \geq x + b 
\end{cases}
\]

so we must have $x + b = x/2 + 1/4 \iff x = 1/2 - 2b$

There is a 2-partitional equilibrium if and only if $b \leq 1/4$
The interval $[x, 1]$ is $4b$ larger than $[0, x]$.

\[ x = \frac{1}{2} - 2b \]
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This can be generalized to $n$-partitional equilibria:
The interval $[x, 1]$ is $4b$ larger than $[0, x]$

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For every $k$, the sender of type $t = x_k$ should be indifferent between sending $m_k$ and $m_{k+1}$
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$$\Rightarrow x_k + b = \frac{x_k-1+x_k}{2} + \frac{x_k+x_k+1}{2} = \frac{x_k-1 + 2x_k + x_k+1}{4}$$
The interval \([x, 1]\) is \(4b\) larger than \([0, x]\)

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\Rightarrow x_k + b = \frac{x_{k-1} + x_k}{2} + \frac{x_k + x_{k+1}}{2} = \frac{x_{k-1} + 2x_k + x_{k+1}}{4}
\]

so \([x_{k+1} - x_k] = [x_k - x_{k-1}] + 4b\)
\[
\Rightarrow x_k = x_1 + (x_1 + 4b) + (x_1 + 2(4b)) + \cdots + (x_1 + (k - 1)(4b)) \\
= kx_1 + (1 + 2 + \cdots + (k - 1))4b = kx_1 + \frac{k(k - 1)}{2}4b
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\(\Rightarrow\) Given \(b\), the largest \(n\) such that there exists a \(n\)-partitional equilibrium is the largest \(n\), denoted by \(n^*(b)\), such that
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\[ 2n(n - 1)b < 1 \]
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\[ 2n(n - 1)b < 1 \iff n^2 - n - 1/2b < 0 \]
\[ \Rightarrow x_k = x_1 + (x_1 + 4b) + (x_1 + 2(4b)) + \cdots + (x_1 + (k-1)(4b)) = kx_1 + (1 + 2 + \cdots + (k-1))4b = kx_1 + \frac{k(k-1)}{2}4b \]

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\[
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\[
2n(n-1)b < 1 \iff n^2 - n - 1/2b < 0
\]

\[
\iff n < \frac{1 + \sqrt{1 + 2/b}}{2}
\]
\[ x_k = x_1 + (x_1 + 4b) + (x_1 + 2(4b)) + \cdots + (x_1 + (k-1)(4b)) \]
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\[ 2n(n-1)b < 1 \iff n^2 - n - 1/2b < 0 \]

\[ \Leftrightarrow n < \frac{1 + \sqrt{1 + 2/b}}{2} = \begin{cases} 2 & \text{if } b = 1/4 \\ +\infty & \text{if } b \to 0 \end{cases} \]
\[ x_k = x_1 + (x_1 + 4b) + (x_1 + 2(4b)) + \cdots + (x_1 + (k - 1)(4b)) \]
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\( \text{☞ A } n\text{-partitional equilibrium exists if } b < \frac{1}{2n(n-1)} \)

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\[ 2n(n - 1)b < 1 \Leftrightarrow n^2 - n - 1/2b < 0 \]
\[ \Leftrightarrow n < \frac{1 + \sqrt{1 + 2/b}}{2} = \begin{cases} 2 & \text{if } b = 1/4 \\ +\infty & \text{if } b \to 0 \end{cases} \]

but full revelation of information is impossible as long as players’ preferences are not perfectly aligned (\( b \neq 0 \))
For which positive values of $b$ does there exist a 3-partitional equilibrium?

Characterize all equilibria when $b = 1/10$

Verify that, in general, the best equilibrium for the expert depends on his type
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Welfare comparison of equilibria

Ex-ante expected utility of the decisionmaker at a $n$-partitional equilibrium:
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**Welfare comparison of equilibria**

Ex-ante expected utility of the decisionmaker at a $n$-partitional equilibrium:

$$EU_2 =$$
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Ex-ante expected utility of the decisionmaker at a \( n \)-partitional equilibrium:

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EU_2 = E \left[ -\left[ \sigma_2(\sigma_1(t)) - t \right]^2 \right]
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**Welfare comparison of equilibria**

Ex-ante expected utility of the decisionmaker at a $n$-partitional equilibrium:

$$EU_2 = E \left[ -[\sigma_2(\sigma_1(t)) - t]^2 \right] = - \int_0^1 [\sigma_2(\sigma_1(t)) - t]^2 dt$$
For which positive values of $b$ does there exist a 3-partitional equilibrium?

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**Welfare comparison of equilibria**

Ex-ante expected utility of the decisionmaker at a $n$-partitional equilibrium:

$$EU_2 = E\left[-\left[\sigma_2(\sigma_1(t)) - t\right]^2\right] = -\int_{0}^{1} [\sigma_2(\sigma_1(t)) - t]^2 dt$$

$$= -\sum_{k=1}^{n} \int_{x_{k-1}}^{x_k} [\sigma_2(m_k) - t]^2 dt$$
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$$= - \sum_{k=1}^{n} \int_{x_{k-1}}^{x_k} \left[ \sigma_2(m_k) - t \right]^2 dt = - \sum_{k=1}^{n} \int_{x_{k-1}}^{x_k} \left[ \frac{x_{k-1} + x_k}{2} - t \right]^2 dt$$
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$$EU_2 = E \left[ -[\sigma_2(\sigma_1(t)) - t]^2 \right] = - \int_0^1 [\sigma_2(\sigma_1(t)) - t]^2 \, dt$$

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$$= - \sum_{k=1}^n \frac{1}{3} \left[ \left( t - \frac{x_{k-1} + x_k}{2} \right) \right]^{x_k} x_{k-1}$$
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$$EU_2 = E \left[ -[\sigma_2(\sigma_1(t)) - t]^2 \right] = -\int_0^1 [\sigma_2(\sigma_1(t)) - t]^2 \, dt$$

$$= -\sum_{k=1}^n \int_{x_{k-1}}^{x_k} [\sigma_2(m_k) - t]^2 \, dt = -\sum_{k=1}^n \int_{x_{k-1}}^{x_k} \left[ \frac{x_{k-1} + x_k}{2} - t \right]^2 \, dt$$

$$= -\sum_{k=1}^n \frac{1}{3} \left[ \left( t - \frac{x_{k-1} + x_k}{2} \right)^3 \right]_{x_{k-1}}^{x_k} = -\frac{1}{12} \sum_{k=1}^n (x_k - x_{k-1})^3$$
\[ x_k - x_{k-1} = \frac{k}{n} - 2kb(n - k) - \left(\frac{(k - 1)}{n} - 2(k - 1)b(n - (k - 1))\right) \]
\[ = 1/n + 2b(2k - n - 1) \]
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\[
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so
\[ EU_2 = -\frac{1}{12} \sum_{k=1}^{n} \left( \frac{1}{n} + 2b (2k - n - 1) \right)^3 \]

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so \( EU_2 = -\frac{1}{12} \sum_{k=1}^{n} \left(\frac{1}{n} + 2b(2k-n-1)\right)^3 \alpha \)

In \( \alpha \), members in \( k \) cancel out with members in \( n-k+1 \), so
\( x_k - x_{k-1} = \frac{k}{n} - 2k b (n-k) - \left( \frac{(k-1)}{n} - 2(k-1) b (n - (k-1)) \right) \)

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\[ x_k - x_{k-1} = \frac{k}{n} - 2k b (n - k) - \left(\frac{(k - 1)}{n} - 2(k - 1) b (n - (k - 1))\right) \]

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After some simplifications, using \( \sum_{1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \), we get

\[ EU_2 = -\frac{1}{12n^2} - \frac{b^2(n^2 - 1)}{3} \]
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In \( \alpha \), members in \( k \) cancel out with members in \( n - k + 1 \), so

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\[ \Rightarrow \text{ With a fixed } n, \text{ the expected payoff of the decisionmaker decreases with } b \]
Ex-ante expected utility of the expert at a $n$-partitional equilibrium:
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Ex-ante expected utility of the expert at a \( n \)-partitional equilibrium:

\[
EU_1 = E \left[ -\sigma_2(\sigma_1(t)) - t - b \right]^2
\]
Ex-ante expected utility of the expert at a $n$-partitional equilibrium:

$$EU_1 = E \left[ -[\sigma_2(\sigma_1(t)) - t - b]^2 \right]$$

$$= - \sum_{k=1}^{n} \int_{x_{k-1}}^{x_k} \left[ \frac{x_{k-1} + x_k}{2} - t - b \right]^2 dt$$
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$$= - \sum_{k=1}^{n} \left( \int_{x_{k-1}}^{x_k} \left[ \frac{x_{k-1} + x_k}{2} - t \right]^2 dt + \int_{x_{k-1}}^{x_k} b^2 dt \right)$$

$$- 2b \left( \int_{x_{k-1}}^{x_k} \left[ \frac{x_{k-1} + x_k}{2} - t \right] dt \right) \underbrace{0}_{0}$$
Ex-ante expected utility of the expert at a $n$-partitional equilibrium:

$$EU_1 = E \left[-[\sigma_2(\sigma_1(t)) - t - b]^2\right]$$

$$= - \sum_{k=1}^{n} \int_{x_{k-1}}^{x_k} \left[\frac{x_{k-1} + x_k}{2} - t - b\right]^2 dt$$

$$= - \sum_{k=1}^{n} \left( \int_{x_{k-1}}^{x_k} \left[\frac{x_{k-1} + x_k}{2} - t\right]^2 dt + \int_{x_{k-1}}^{x_k} b^2 dt - 2b \int_{x_{k-1}}^{x_k} \left[\frac{x_{k-1} + x_k}{2} - t\right] dt \right)$$

so $EU_1 = EU_2 - b^2$ is also decreasing with $b$ when $n$ is fixed
Which equilibrium is the most efficient?
Which equilibrium is the most efficient?

- We compare $EU_2$ (or $EU_1$) at a $n$-partitional equilibrium with $EU_2$ (or $EU_1$) at a $(n-1)$-partitional equilibrium:
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After some simplifications we find, for every $n \geq 1$, $EU_2[n] - EU_2[n - 1] > 0$ if and only if

$$b < \frac{1}{2n(n - 1)}$$
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which is exactly the existence condition for a $n$-partitional equilibrium.
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which is exactly the existence condition for a $n$-partitional equilibrium

**Remark** If information could be transmitted credibly, then the expected payoffs of both players would be higher than in all equilibria since we would have $EU_2 = 0$ and $EU_1 = -b^2$. We will see that the same outcome is achieved with certifiable information
Generalization
Generalization

All equilibria are partitional equilibria, and $n$-partitional equilibria exist for increasing values of $n$ when players’ conflict of interest decrease, in a larger class of games:
Generalization

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- Types of the expert: $T$, distribution $F(t)$ with density $f(t)$
Generalization

All equilibria are partitional equilibria, and \( n \)-partitional equilibria exist for increasing values of \( n \) when players’ conflict of interest decrease, in a larger class of games:

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In general, equilibria cannot be compared in terms of efficiency anymore
Variations and Extensions
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But this is not general. In Example 3, if cost(a) = 3 ∀ k then a FRE exists (k₁ → a and k₂ → b) while cheap talk is not credible.

In this example, strategic money burning improves Pareto efficiency. The same phenomenon is possible in Crawford and Sobel’s model (see Austen-Smith and Banks, 2000, 2002).
• **Cheap Talk vs. Delegation**
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Alternative to communication: the decisionmaker delegates the decision \( a \in [0, 1] \) to the expert

Example: in a firm, instead of collecting all the information from the different hierarchical levels of the organization, a manager may delegate some decisions (e.g., investment decisions) to agents in lower levels of the hierarchies, even if these agents do not have exactly same incentives as the manager
Delegation of the decision to the expert $\Rightarrow$ action $a_1^*(t) = t + b$ is chosen when the expert’s type is $t$.
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In the cheap talk game:

$$\begin{cases} EU_1 = EU_2 - b^2 \\ EU_2 = -\frac{1}{12n^2} - \frac{b^2(n^2-1)}{3} \end{cases}$$

where $n$ is such that $b \leq \frac{1}{2n(n-1)}$
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Of course, the expert always prefers delegation. The DM prefers delegation to cheap talk if $EU_2^D \geq EU_2 \Leftrightarrow b^2 \leq \frac{1}{12n^2} + \frac{b^2(n^2-1)}{3}$
Hence, delegation is optimal if $b \leq 1/4$ and $n \geq 2$ or $b \leq 1/\sqrt{12} \simeq 1/3.5$ and $n = 1$. 
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With an extreme bias ($b > 1/3.5$) the decision maker plays the optimal action of the silent game $a = E[t] = 1/2$ (no delegation, no informative communication).
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$\Rightarrow$ Delegation of the decision right is often preferred over cheap talk because the welfare loss caused by self-interested communication is higher than costs of biased decision-making
Hence, delegation is optimal if \( b \leq 1/4 \) and \( n \geq 2 \) or \( b \leq 1/\sqrt{12} \approx 1/3.5 \) and \( n = 1 \).

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\( \Rightarrow \) Delegation of the decision right is often preferred over cheap talk because the welfare loss caused by self-interested communication is higher than costs of biased decision-making.

Dessein (2002) shows more generally (for a non-uniform prior distribution) that delegation is better than communication, except when the expert has a small informational advantage and communication is very noisy.
• **Cheap Talk vs. Commitment.**
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Consider a mechanism design / principal-agent approach, but without transfers (Melumad and Shibano, 1991)
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The decisionmaker (the principal) commits to a decision rule

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\max_{a(\cdot)} - \int_0^1 (a(t) - t)^2 \, dt \\
\text{u.t.c.} \quad -(a(t) - t - b)^2 \geq -(a(t') - t - b)^2, \quad \forall t, t' \in T.
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Of course, if \( b \neq 0 \), the (first best) decision rule \( a(t) = t \) does not satisfy the informational incentive constraint
The informational incentive constraint implies

\[ a'(t)(a(t) - t - b) = 0, \quad \forall t \in T, \]

so on every interval \( a(t) \) is either constant or \( a(t) = t + b = a_1^*(t) \). In particular, full separation, \( a(t) = t + b \), and full bunching, \( a(t) = a \), satisfy the constraint.
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\[
a(t) = \begin{cases} 
  t_1 + b & \text{if } t \leq t_1, \\
  t + b & \text{if } t \in [t_1, t_2], \\
  t_2 + b & \text{if } t \geq t_2,
\end{cases}
\]

or should be constant on \( T \).
Hence, the principal minimizes

\[
\int_0^{t_1} (a(t) - t)^2 \, dt + \int_{t_1}^{t_2} (a(t) - t)^2 \, dt + \int_{t_2}^1 (a(t) - t)^2 \, dt
\]

\[
= - \left( \frac{1}{3} \right) (b^3 - (t_1 + b)^3) + b^2(t_2 - t_1) - \left( \frac{1}{3} \right)((t_2 + b - 1)^3 - b^3),
\]

if \(0 \leq t_1 \leq t_2 \leq 1\), or chooses \(a(t) = 1/2\) for all \(t\)
The solution is \((t_1, t_2) = (0, 1 - 2b)\) if \(b \leq 1/2\), and \(a(t) = 1/2\) for all \(t \in T\) if \(b \geq 1/2\)
The solution is $(t_1, t_2) = (0, 1 - 2b)$ if $b \leq 1/2$, and $a(t) = 1/2$ for all $t \in T$ if $b \geq 1/2$.
Comparing cheap talk, delegation ($D$) and commitment ($C$), we have:

\[ EU_1^D \geq EU_1^C \geq EU_1, \]
\[ EU_2^C \geq EU_2^D \geq EU_2 \]

⇒ The best situation for the decisionmaker is commitment (contracting) and that of the expert, delegation. Whatever the equilibrium, cheap talk communication is always worse than delegation and commitment for both players.
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\(\Rightarrow\) The best situation for the decisionmaker is commitment (contracting) and that of the expert, delegation. Whatever the equilibrium, cheap talk communication is always worse than delegation and commitment for both players.

**Remark.** The optimal mechanism can be implemented with a delegation set \(D = [0, 1-b]\), the principal letting the agent choose any action in \(D\).
• Multiple Senders and Multidimensional Cheap Talk

Usual models of cheap talk: unidimensional policy decision and information
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Basic insight: information transmission decreases when the conflict of interest between the interested parties (the senders) and the decisionmaker (the receiver) increases
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For all $i \in \{1, 2, p\}$, $u_i(x, \theta)$ is continuous and quasi concave in $x$
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Ideal points: $\theta + x_i \in \mathbb{R}^d$, where $x_p = 0$
Assume quadratic utilities:

\[ u_i(x, \theta) = -\sum_{j=1}^{d} (x^j - (x^j_i + \theta))^2 \]
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Timing:

① Nature chooses \( \theta \)
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DM's belief: $\mu : \mathcal{M} \times \mathcal{M} \rightarrow \Delta(\Theta)$
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DM's strategy: \( x : M \times M \rightarrow \mathbb{R}^d \)

**Fully Revealing Equilibrium (FRE):**

\[ \mu(\theta \mid s_1(\theta), s_2(\theta)) = 1, \quad \text{for all } \theta \in \Theta \]
Unidimensional Case.
Unidimensional Case.

• Gilligan and Krehbiel (1989, American Journal of Political Science)

• Krishna and Morgan (2001, QJE)

• Battaglini (2002, Ecta)
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A FRE may exist if experts’ ideal points are not too extreme
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- E.g., when $x_1, x_2 > 0$, there is a FRE $s_1(\theta) = s_2(\theta) = \theta$ with

$$x(s_1(\theta), s_2(\theta)) = \min\{s_1(\theta), s_2(\theta)\}$$
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  \]

- When \( x_1 < 0 < x_2 \), a FRE exists if \( |x_1| + |x_2| \) is not too large, but may rely on implausible (extreme) beliefs off the equilibrium path
Multidimensional Case.
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**Proposition 1 (Battaglini, 2002)** If \( d = 2 \) and \( x_1 \neq \alpha x_2 \) for all \( \alpha \in \mathbb{R} \) (i.e., \( x_1 \) and \( x_2 \) and linearly independent), then there is a FRE
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Proof.

Each expert will reveal the tangent of the other expert’s indifference curve at the DM’s ideal point \( \theta \)
Strategic Information Transmission: Cheap Talk Games

\( l_1(\theta) \)

\( l_2(\theta) \)

\( \theta + x_2 \)

\( \theta + x_1 \)

\( \theta \)
Let

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The following strategy profile and beliefs constitute a FR PBE:

- \( s_i(\theta) = l_j(\theta), \ i \neq j \)
- \( \mu(s_1, s_2) = s_1 \cap s_2 \) (and any point in \( l_i(\theta) \) if \( s_i \cap l_i(\theta) = \emptyset \))
- \( x(s_1, s_2) = \mu(s_1, s_2) \)
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If expert \( i \) reveals \( \hat{s}_i \) when the state is \( \theta \), then the action of the DM is

\[
x(\hat{s}_i, s_j(\theta)) = \mu(\hat{s}_i, l_i(\theta)) \in l_i(\theta)
\]

which, by construction, is the closest to \( i \)'s ideal point when \( \hat{s}_i = l_j(\theta) \) □
**Remark** The result can be extended to more than two dimensions of the policy space and to quasi-concave utilities (not necessarily quadratic), but may not be robust to the timing of the game (sequential cheap talk)
• **Lobbying with Several Audiences.** Farrell and Gibbons (1989) show in a model with two decisionmakers that the expert’s announcement may be more credible when communication takes place publicly.
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Example:

<table>
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<tr>
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There exists a fully revealing equilibrium when the lobbyist communicates *privately* with the decisionmaker $Q$ ($R$, respectively) if and only if $v_1 \geq 0$ and $v_2 \geq 0$ ($w_1 \geq 0$ and $w_2 \geq 0$, respectively).
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There exists a fully revealing equilibrium when the lobbyist communicates privately with the decisionmaker Q (R, respectively) if and only if \( v_1 \geq 0 \) and \( v_2 \geq 0 \) (\( w_1 \geq 0 \) and \( w_2 \geq 0 \), respectively).

There exists a fully revealing equilibrium when the lobbyist communicates publicly with the two decisionmakers if and only if \( v_1 + w_1 \geq 0 \) and \( v_2 + w_2 \geq 0 \).
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There exists a fully revealing equilibrium when the lobbyist communicates publicly with the two decisionmakers if and only if $v_1 + w_1 \geq 0$ and $v_2 + w_2 \geq 0$.

Mutual discipline: There is no separating equilibrium in private, but there is in public. E.g., when $v_1 = w_2 = 3$ and $v_2 = w_1 = -1$. 
Some Experimental Evidence
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Dickhaut et al. (1995, ET).

- Crawford and Sobel (1982) with 4 states and 4 actions
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- Five treatments (biases)

\[ b_1, b_2, b_3, b_4, b_5 \]

- \( F \text{RE, } P \text{RE, } N \text{RE} \)
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\[ \begin{align*}
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\end{align*} \]

- 12 repetitions among 8 subjects with random matching
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Results:

- Observed average distance between states and actions increases with the bias \( b \)
- Receivers’ average payoffs decrease with \( b \)
- Two much information is revealed when it should not (\( b_4, b_5 \))

- Crawford and Sobel (1982) with 5 states and 9 actions

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- Four treatments (biases) with the most informative equilibria being

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\[
\begin{align*}
\{b_1, b_2, b_3, b_4\} \\
\{FRE, PRE_1, PRE_2, NRE\}
\end{align*}
\]

Results:

- Observed correlation between
  - states and actions
  - messages and actions
  - states and messages
- decreases with the bias \( b \)
• Receivers’ and Senders’ average payoffs decrease with $b$, and are consistent with the most informative equilibrium.
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Seller-Buyer relationship, where the seller knows the asset quality
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The unique communication equilibrium is non-revealing (monotonic game).
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  - efficiency is significantly higher than predicted
  - but at the expense of buyers (they overpay by relying on sellers’ exaggerated claims)
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- Without communication possibility, actual efficiency close to theoretical efficiency
- With cheap talk communication, the adverse selection problem is not as severe as predicted
  - efficiency is significantly higher than predicted
  - but at the expense of buyers (they overpay by relying on sellers’ exaggerated claims)
- With certifiable information,
  - efficiency is smaller than predicted, but higher than under cheap talk
  - no wealth transfer from buyers to sellers anymore
References


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