Cheap Talk Games: Extensions

Outline
(November 12, 2008)

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- The Art of Conversation: Multistage Communication and Compromises
- Mediated Communication: Correlated and Communication Equilibria

1 The Art of Conversation: Multistage Communication and Compromises

Aumann et al. (1968): Allowing more than one communication stage can extend and Pareto improve the set of Nash equilibria, even if only one player is privately informed

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Aumann and Hart (2003, Ecta): Full characterization of equilibrium payoffs induced by multistage cheap talk communication in finite two-player games with incomplete information on one side

Multistage communication also extends the equilibrium outcomes in the classical model of Crawford and Sobel (1982)

1.1 Examples

Example. (Compromising)

	L	R
T	$6,2$	$0,0$
B	$0,0$	$2,6$

Jointly controlled lottery (JCL):

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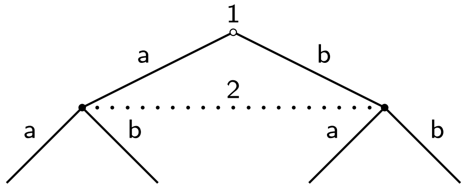

	L	R	L	R	L	R	L	R
T	$\mathbf{6,2}$	$0,0$	$6,2$	$0,0$	$6,2$	$0,0$	$\mathbf{6,2}$	$0,0$
B	$0,0$	$2,6$	$0,0$	$\mathbf{2,6}$	$0,0$	$\mathbf{2,6}$	$0,0$	$2,6$

$$\frac{1}{2}\mathsf{a} + \frac{1}{2}\mathsf{b} \Rightarrow \frac{1}{2}(T,L) + \frac{1}{2}(B,R) \to (4,4)$$

Example. (Signalling, and then compromising)

k_1	L	M	R
T	$(6,2)$	$(0,0)$	$(3,0)$
B	$(0,0)$	$(2,6)$	$(3,0)$

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k_2	L	M	R
T	$(0,0)$	$(0,0)$	$(4,4)$
B	$(0,0)$	$(0,0)$	$(4,4)$

Interim equilibrium payoffs $((4,4),4)$

The two communication stages **cannot** be reversed (compromising should come after signalling)

Example. (Compromising, and then signaling) (Example 5)

	j_1	j_2	j_3	j_4	j_5	
k_1	$1, 10$	$3, 8$	$0, 5$	$3, 0$	$1, -8$	p
k_2	$1, -8$	$3, 0$	$0, 5$	$3, 8$	$1, 10$	$1-p$

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Interim equilibrium payoffs $((2, 2), 8) = \frac{1}{2}((3, 3), 6) + \frac{1}{2}((1, 1), 10)$

Of course, the two communication stages cannot be reversed (the compromise determines the type of signalling)

Example. (Signalling, then compromising, and then signalling)

	j_1	j_2	j_3	j_4	j_5	j_6	
k_1	$1, 10$	$3, 8$	$0, 5$	$3, 0$	$1, -8$	$2, 0$	$1/3$
k_2	$1, -8$	$3, 0$	$0, 5$	$3, 8$	$1, 10$	$2, 0$	$1/3$
k_3	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$2, 8$	$1/3$

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Interim equilibrium payoffs $((2, 2, 2), 8)$

1.2 Multistage and Bilateral Cheap Talk Game $\Gamma_n^0(p)$

Bilateral communication: the uninformed player can also send messages

Player 1: informed, expert

Player 2: uninformed, decision maker

K: set of information states (i.e., types) of P1, probability distribution p

7/ J: set of actions of P2

P1's payoff is $A^k(j)$ and P2's payoff is $B^k(j)$

M^1 : set of messages of the expert (independent of his type)

M^2 : set of message of the decisionmaker

At every stage $t = 1, \ldots, n$, P1 sends a message $m_t^1 \in M^1$ to P2 and, simultaneously, P2 sends a message $m_t^2 \in M^2$ to P1

At stage $n + 1$, P2 chooses j in J

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Information Phase	Communication Phase	Action Phase
Expert learns $k \in K$	Expert and DM send $(m_t^1, m_t^2) \in M^1 \times M^2$ $(t = 1, \ldots n)$	DM chooses $j \in J$

1.3 Characterization of the Nash equilibria of $\Gamma_n^0(p), n = 1, 2, \ldots$

Hart (1985), Aumann and Hart (2003): finite case (K and J are finite sets)

All Nash equilibrium payoffs of the multistage, bilateral communication games $\Gamma_n^0(p)$, $n = 1, 2, \ldots$, are characterized geometrically from the graph of the equilibrium correspondence of the silent game

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Additional stages of cheap talk can Pareto-improve the equilibria of the communication game (**Aumann et al., 1968**)

Imposing no deadline to cheap talk can Pareto-improve the equilibria of any n-stage communication game (**Forges, 1990b, QJE, Simon, 2002, GEB**)

Example. (Forges, 1990a, QJE)

An employer (the DM) chooses to offer a job j_1, j_2, j_3 or j_4, or no job (action j_0) to a candidate (the expert)

The candidate has two possible types k_1 et k_2, which determine his competence and preference for the different jobs

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	j_1	j_2	j_0	j_3	j_4	
k_1	$6, 10$	$10, 9$	$0, 7$	$4, 4$	$3, 0$	$\Pr[k_1] = p$
k_2	$3, 0$	$4, 4$	$0, 7$	$10, 9$	$6, 10$	$\Pr[k_2] = 1 - p$

$$Y(p) = \begin{cases} \{j_1\} & \text{if } p > 4/5, \\ \{j_2\} & \text{if } p \in (3/5, 4/5), \\ \{j_0\} & \text{if } p \in (2/5, 3/5), \\ \{j_3\} & \text{if } p \in (1/5, 2/5), \\ \{j_4\} & \text{if } p < 1/5. \end{cases}$$

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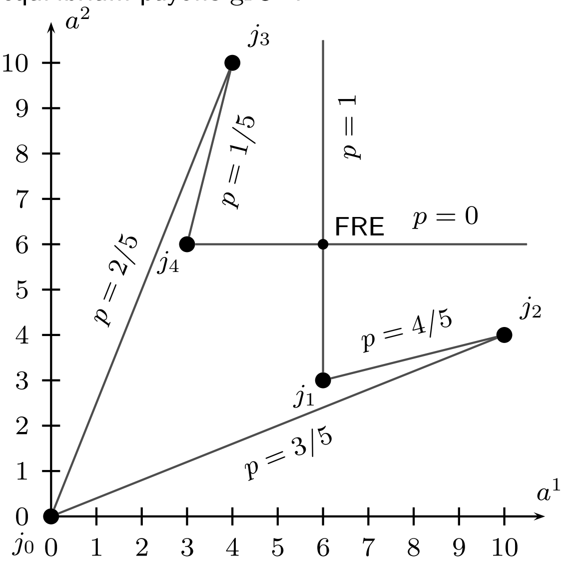
Graph of modified equilibrium payoffs $\mathrm{gr}\,\mathcal{E}^{+}$:



From the equilibrium characterization theorem for $\Gamma^0_S(p)$, there is only two types of equilibria in the single-stage cheap talk game: NRE and FRE

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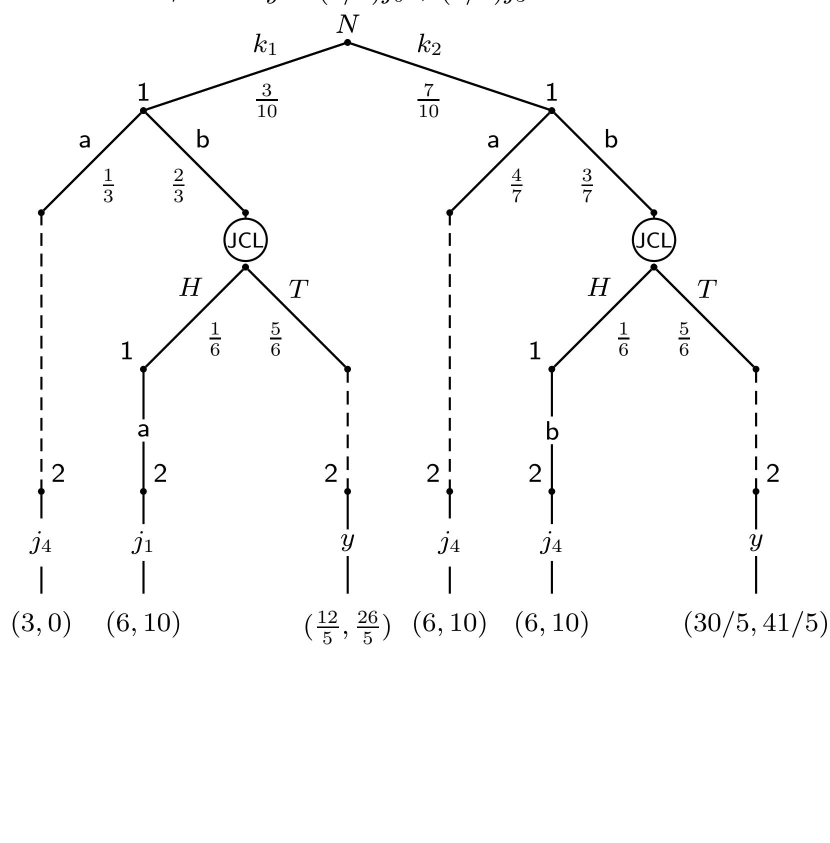
But in the 3-stage cheap talk game, when $p = 3/10$, the interim payoff $(3, 6)$ can be obtained as follows, where $y = (2/5)j_0 + (3/5)j_3$



Geometrically, this equilibrium payoff can be constructed as follows

Adding a JCL before the one-stage cheap talk game at $p = 1/5$ yields $[j_3, j_4, \text{FRE}]$

Adding a JCL before the one-stage cheap talk game at $p = 2/5$ yields $[j_0, j_3, \text{FRE}]$

Adding a signalling stage before the JCL allows a second convexification at p fixed

Hence, for all $p \in [1/5, 2/5]$ (in particular, $p = 3/10$) we get $[j_3, j_4, \text{FRE}]$ (in particular, $a = (3, 6)$) with three communication stages

A subset of $\mathbb{R}^2 \times \mathbb{R} \times [0, 1]$ is *diconvex* if it is convex in (β, p) when a is fixed, and convex in (a, β) when p is fixed. di-co$\,(E)$ is the smallest diconvex set containing E

Theorem. (Hart, 1985, Forges, 1994, Aumann and Hart, 2003) Let $p \in (0, 1)$. A payoff (a, β) is an equilibrium payoff of some bilateral communication game $\Gamma_n^0(p)$, for some length n, if and only if (a, β, p) belongs to di-co$\,(\mathrm{gr}\,\mathcal{E}^+)$, the set of all points obtained by diconvexifying the set $\mathrm{gr}\,\mathcal{E}^+$

1.4 Communication with No Deadline

When the number of communication stages, n, is not fixed in advance, the job candidate can even achieve the expected payoff $(7, 7)$ when $p = 1/2$

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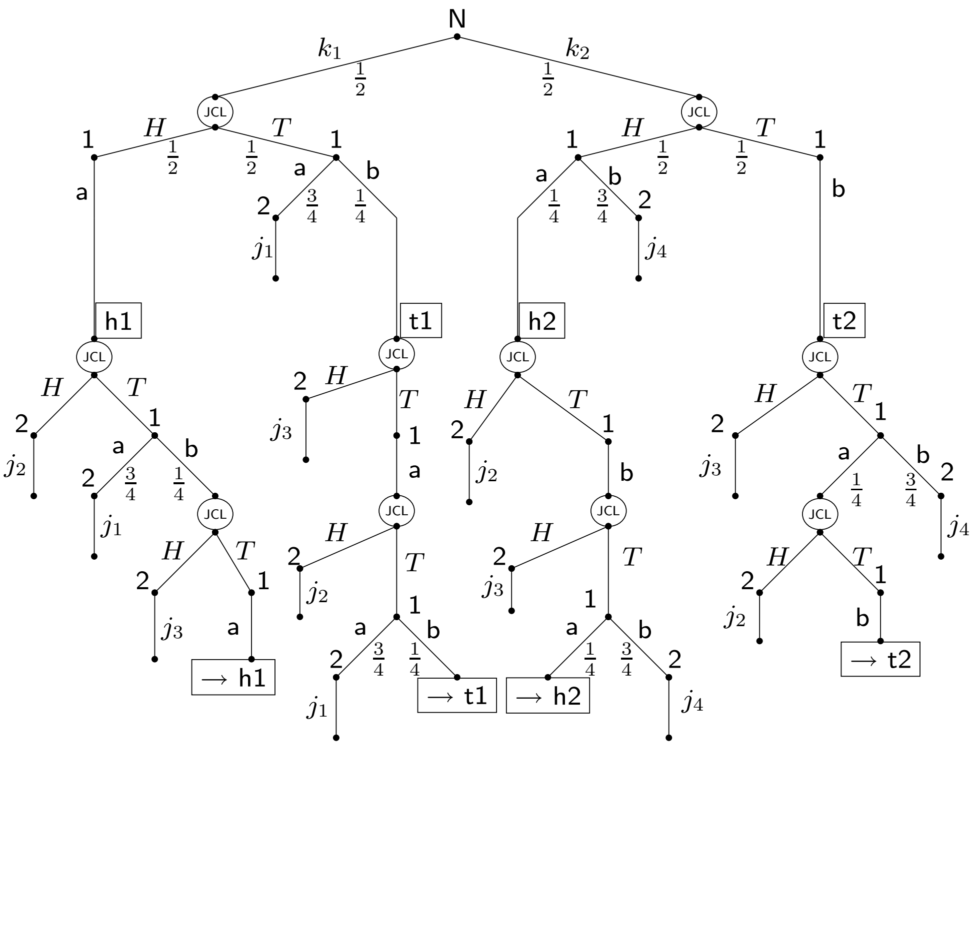

1.5 Conversation in Crawford and Sobel's Model

Krishna and Morgan (2004, JET): In the model of Crawford and Sobel (1982), adding several bilateral communication stages can Pareto-improve all the equilibria of the unilateral cheap talk game

Configuration 1: Intermediate Bias ($b = 1/10$).

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When $b = 1/10$, there is two possible types of equilibria in the model of Crawford and Sobel: a NRE and a 2-partitional equilibrium

The 2-partitional equilibrium is the most efficient one, and is given by

$$\sigma_1(t) = \begin{cases} m_1 & \text{if } t \in [0, x) \\ m_2 & \text{if } t \in [x, 1], \end{cases}$$

where $x = 1/2 - (2/10)(2-1) = 3/10$, $\sigma_2(m_1) = x/2 = 3/20$, $\sigma_2(m_2) = (1+x)/2 = 13/20$

and

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$$EU_2 = -\frac{1}{12 \times 2^2} - \frac{(1/10)^2(2^2-1)}{3} = -37/1200$$
$$EU_1 = EU_2 - b^2 = -49/1200$$

The following equilibrium in the 3-stage game is (ex-ante) Pareto improving

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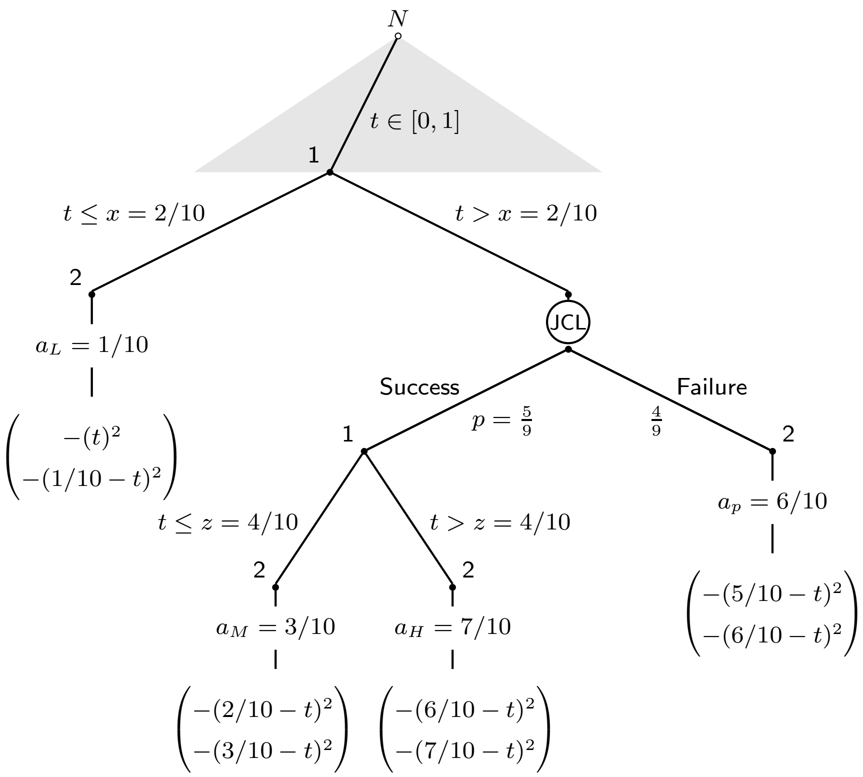

Ex-ante expected payoffs:

$$
\begin{aligned}
EU_1 &= -\int_0^{2/10} t^2\,dt - \frac{5}{9}\left[\int_{2/10}^{4/10} (2/10 - t)^2\,dt + \int_{4/10}^{1} (6/10 - t)^2\,dt\right] \\
&\quad - \frac{4}{9}\int_{2/10}^{1} (5/10 - t)^2\,dt = -\frac{48}{1200}
\end{aligned}
$$

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$$
EU_2 = EU_1 + b^2 = -\frac{36}{1200}
$$

Configuration 2: High Bias ($b = 7/24$).

When $b = 7/24 > 1/4$ the unique equilibrium with unilateral communication in NR

The following (non-monotonic) equilibrium of the 3-stage game, where $x = 0.048$ and $z = 0.968$, Pareto dominates this NRE

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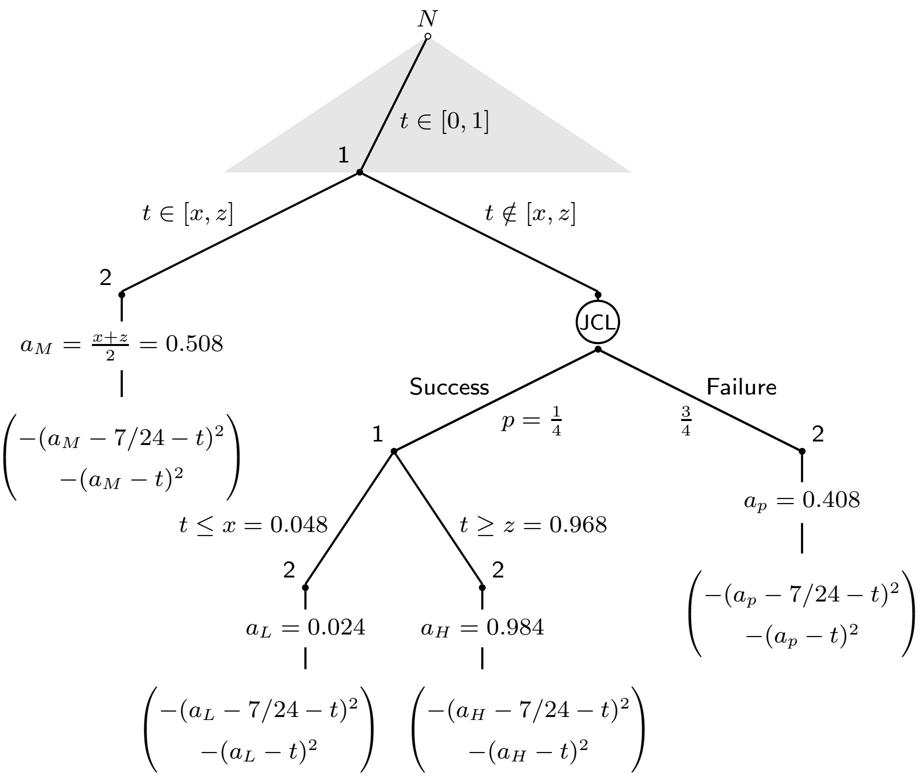

More generally, Krishna and Morgan (2004) show that

- for all $b < 1/8$, there is a monotonic Nash equilibrium outcome of the 3-stage communication game which Pareto dominates all equilibrium outcomes of the unilateral communication game (Krishna and Morgan, 2004, Theorem 1)
- for all $b \in (1/8, 1/\sqrt{8})$, there is a non-monotonic Nash equilibrium outcome of the 3-stage communication game which Pareto dominates the unique NR equilibrium outcome of the unilateral communication game
23/ (Krishna and Morgan, 2004, Theorem 2)
- for all $b > 1/8$ it is not possible to Pareto improve the unique NR equilibrium outcome of the unilateral communication game with monotonic equilibria Krishna and Morgan (2004, Proposition 3)

2 Mediated Communication: Correlated and Communication Equilibria

2.1 Complete Information Games: Correlated Equilibrium

What is the set of all equilibrium payoffs that can be achieved in a normal form
24/ game when we allow any form of preplay communication (including possibly mediated communication)?

At least, players are able to achieve the convex hull of the set of Nash equilibrium payoffs, by using jointly controlled lotteries, or simply by letting a mediator publicly reveal the realization of a random device

For example, tossing a fair coin allows to achieve the outcome $\mu = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$ with payoffs $(\frac{9}{2}, \frac{9}{2})$ in the chicken game:

	a	b
a	$(2,7)$	$(6,6)$
b	$(0,0)$	$(7,2)$

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More generally, adding any system for preplay communication generates some information system

$$\langle \Omega,\, p,\, (\mathcal{P}_i)_{i \in N} \rangle$$

so a Nash equilibrium of this extended game exactly corresponds to

Definition (Aumann, 1974) A **correlated equilibrium** (CE) of the normal form game

$$\langle N,\, (A_i)_{i \in N},\, (u_i)_{i \in N} \rangle$$

26/ is a pure strategy Nash equilibrium of the Bayesian game

$$\langle N,\, \Omega,\, p,\, (\mathcal{P}_i)_i,\, (A_i)_i,\, (u_i)_i \rangle$$

where $u_i(a; \omega) = u_i(a)$, i.e., a profile of pure strategies $s = (s_1, \ldots, s_n)$ such that for every $i \in N$ and every strategy r_i of player i:

$$\sum_{\omega \in \Omega} p(\omega)\ u_i(s_i(\omega), s_{-i}(\omega)) \geq \sum_{\omega \in \Omega} p(\omega)\ u_i(r_i(\omega), s_{-i}(\omega))$$

➥ *Correlated equilibrium outcome* $\mu \in \Delta(A)$, where

$$\mu(a) = p(\{\omega \in \Omega : s(\omega) = a\})$$

➥ *Correlated equilibrium payoff* $\sum_{a \in A} \mu(a) u_i(a)$, $i = 1, \ldots, n$

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The set of CE outcomes may be strictly larger than the convex hull of Nash equilibrium outcomes

$$\mathcal{P}_1 = \{\underbrace{\{\omega_1, \omega_2\}}_{a}, \underbrace{\{\omega_3\}}_{b}\}$$

$$\mathcal{P}_2 = \{\underbrace{\{\omega_1\}}_{a}, \underbrace{\{\omega_2, \omega_3\}}_{b}\}$$

	a	b
a	$(2,7)$	$(6,6)$
b	$(0,0)$	$(7,2)$

➡ Correlated equilibrium payoff $(5,5) \notin \mathrm{co}\{\text{EN}\}$

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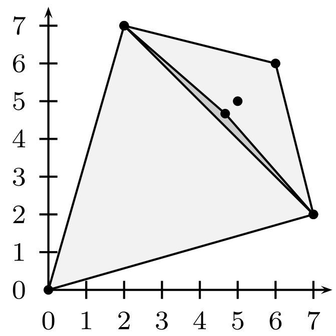

A correlated equilibrium can Pareto dominate every Nash equilibrium

The following game, where $z^+ = z + \varepsilon$ and $z^- = z - \varepsilon$

$$\begin{pmatrix} 0,0 & x^+, y^- & x^-, y^+ \\ x^-, y^+ & 0,0 & x^+, y^- \\ x^+, y^- & x^-, y^+ & 0,0 \end{pmatrix}$$

has a unique Nash equilibrium

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$$\begin{pmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{pmatrix} \text{ with payoffs } (\frac{2}{3}x, \frac{2}{3}y)$$

while there is a correlated equilibrium

$$\begin{pmatrix} 0 & 1/6 & 1/6 \\ 1/6 & 0 & 1/6 \\ 1/6 & 1/6 & 0 \end{pmatrix} \text{ with payoffs } (x, y)$$

"Revelation principle" for complete information games:

Every correlated equilibrium outcome, i.e., every Nash equilibrium of some preplay communication extension of the game, can be achieved with a mediator who makes private recommendations to the players, and no player has an incentive to deviate from the mediator's recommendation

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Proposition 1 *Every correlated equilibrium outcome of a normal form game* $\langle N, (A_i)_{i\in N}, (u_i)_{i\in N}\rangle$ *is a* **canonical correlated equilibrium** *outcome, where the information structure and strategies are given by:*

- $\Omega = A$
- $\mathcal{P}_i = \{\{a \in A : a_i = b_i\} : b_i \in A_i\}$ *for every* $i \in N$
- $s_i(a) = a_i$ *for every* $a \in A$ *and* $i \in N$

2.2 Incomplete Information Games: Communication Equilibrium

A **communication equilibrium** of a Bayesian game is a Nash equilibrium of some preplay and interim communication extension of the game

- The communication system should possibly include a mediator who can send outputs but also receive **inputs** from the players (two-way communication)

31/ • A communication equilibrium outcome is a mapping $\mu : T \to \Delta(A)$

A **canonical communication equilibrium** of a Bayesian game is a Nash equilibrium of the one-stage communication extension of the game in which each player

- first, truthfully reveals his type to the mediator
- then, follows the recommendation of action of the mediator

i.e. for all $i \in N$, $t_i \in T_i$, $s_i \in T_i$ and $\delta : A_i \to A_i$,

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$$\sum_{t_{-i} \in T_{-i}} \sum_{a \in A} p(t_{-i} \mid t_i) \mu(a \mid t) u_i(a, t) \geq$$

$$\sum_{t_{-i} \in T_{-i}} \sum_{a \in A} p(t_{-i} \mid t_i) \mu(a \mid t_{-i}, s_i) u_i(a_{-i}, \delta(a_i), t)$$

Revelation Principle for Bayesian Games: The set of communication equilibrium outcomes coincides with the set of canonical communication equilibrium outcomes

Example. The geometric characterization theorem shows that face-to-face (even multistage) communication cannot matter in the following game:

	j_1	j_2	j_3
k_1	$3,3$	$1,2$	$0,0$
k_2	$2,0$	$3,2$	$1,3$

33/ But mediated or noisy communication allows some (Pareto improving) information transmission

For example, when $\Pr(k_1) = 1/2$

$$\mu(k_1) = \frac{1}{2}j_1 + \frac{1}{2}j_2 \quad \text{and} \quad \mu(k_2) = j2$$

is a Pareto improving communication equilibrium

Mediation in the (quadratic) model of Crawford and Sobel (1982)

Goltsman et al. (2007): (Face-to-face) cheap talk is as efficient as mediated communication if and *only if* the bias b does not exceed $1/8$

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