Cheap Talk Games: Extensions

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Outline

(November 12, 2008)

- The Art of Conversation: Multistage Communication and Compromises
- Mediated Communication: Correlated and Communication Equilibria

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Multistage communication also extends the equilibrium outcomes in the classical model of Crawford and Sobel (1982)

1.1 Examples

Example. (Compromising)

	L	R
T	$6, 2$	$0, 0$
B	$0, 0$	$2, 6$

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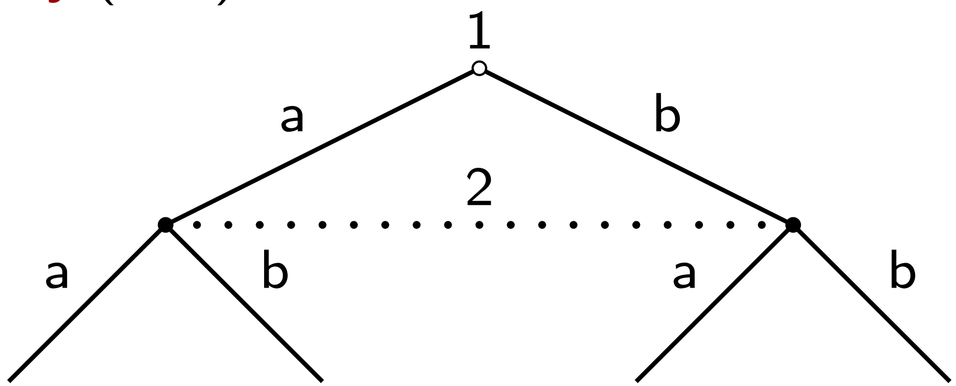

	L	R	L	R	L	R	L	R
T	$\mathbf{6,2}$	$0,0$	$6,2$	$0,0$	$6,2$	$0,0$	$\mathbf{6,2}$	$0,0$
B	$0,0$	$2,6$	$0,0$	$\mathbf{2,6}$	$0,0$	$\mathbf{2,6}$	$0,0$	$2,6$

$$\frac{1}{2}\mathsf{a} + \frac{1}{2}\mathsf{b} \Rightarrow \frac{1}{2}(T,L) + \frac{1}{2}(B,R) \rightarrow (4,4)$$

Example. (Signalling, and then compromising)

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k_1	L	M	R
T	$(6,2)$	$(0,0)$	$(3,0)$
B	$(0,0)$	$(2,6)$	$(3,0)$

k_2	L	M	R
T	$(0,0)$	$(0,0)$	$(4,4)$
B	$(0,0)$	$(0,0)$	$(4,4)$

Interim equilibrium payoffs $((4,4),4)$

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The two communication stages **cannot** be reversed (compromising should come after signalling)

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	j_1	j_2	j_3	j_4	j_5	
k_1	$1,10$	$3,8$	$0,5$	$3,0$	$1,-8$	p
k_2	$1,-8$	$3,0$	$0,5$	$3,8$	$1,10$	$1-p$

Interim equilibrium payoffs $((2,2),8) = \frac{1}{2}((3,3),6) + \frac{1}{2}((1,1),10)$

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Interim equilibrium payoffs $((2,2),8) = \frac{1}{2}((3,3),6) + \frac{1}{2}((1,1),10)$

Of course, the two communication stages cannot be reversed (the compromise determines the type of signalling)

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	j_1	j_2	j_3	j_4	j_5	j_6	
k_1	$1, 10$	$3, 8$	$0, 5$	$3, 0$	$1, -8$	$2, 0$	$1/3$
k_2	$1, -8$	$3, 0$	$0, 5$	$3, 8$	$1, 10$	$2, 0$	$1/3$
k_3	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$2, 8$	$1/3$

Interim equilibrium payoffs $((2, 2, 2), 8)$

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M^1 : set of messages of the expert (independent of his type)

M^2 : set of message of the decisionmaker

At every stage $t = 1, \ldots, n$, P1 sends a message $m_t^1 \in M^1$ to P2 and, simultaneously, P2 sends a message $m_t^2 \in M^2$ to P1

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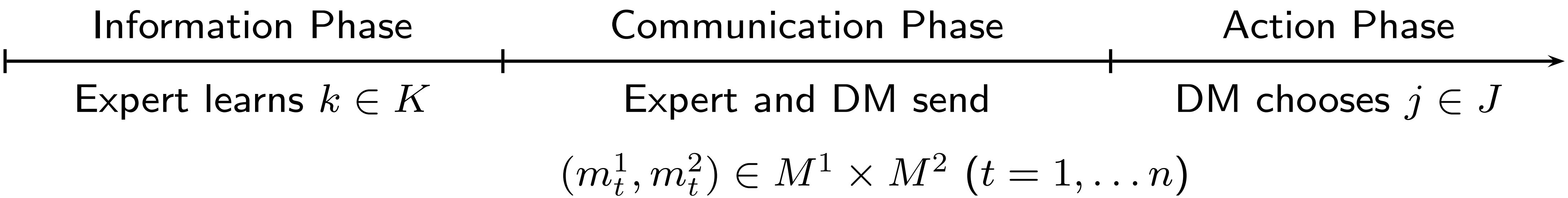

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Additional stages of cheap talk can Pareto-improve the equilibria of the communication game (**Aumann et al., 1968**)

Imposing no deadline to cheap talk can Pareto-improve the equilibria of any n-stage communication game (**Forges, 1990b, QJE, Simon, 2002, GEB**)

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An employer (the DM) chooses to offer a job j_1, j_2, j_3 or j_4, or no job (action j_0) to a candidate (the expert)

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	j_1	j_2	j_0	j_3	j_4	
k_1	$6, 10$	$10, 9$	$0, 7$	$4, 4$	$3, 0$	$\Pr[k_1] = p$
k_2	$3, 0$	$4, 4$	$0, 7$	$10, 9$	$6, 10$	$\Pr[k_2] = 1 - p$

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k_2	$3, 0$	$4, 4$	$0, 7$	$10, 9$	$6, 10$	$\Pr[k_2] = 1 - p$

$$Y(p) = \begin{cases} \{j_1\} & \text{if } p > 4/5, \\ \{j_2\} & \text{if } p \in (3/5, 4/5), \\ \{j_0\} & \text{if } p \in (2/5, 3/5), \\ \{j_3\} & \text{if } p \in (1/5, 2/5), \\ \{j_4\} & \text{if } p < 1/5. \end{cases}$$

Graph of modified equilibrium payoffs $\operatorname{gr}\mathcal{E}^{+}$:

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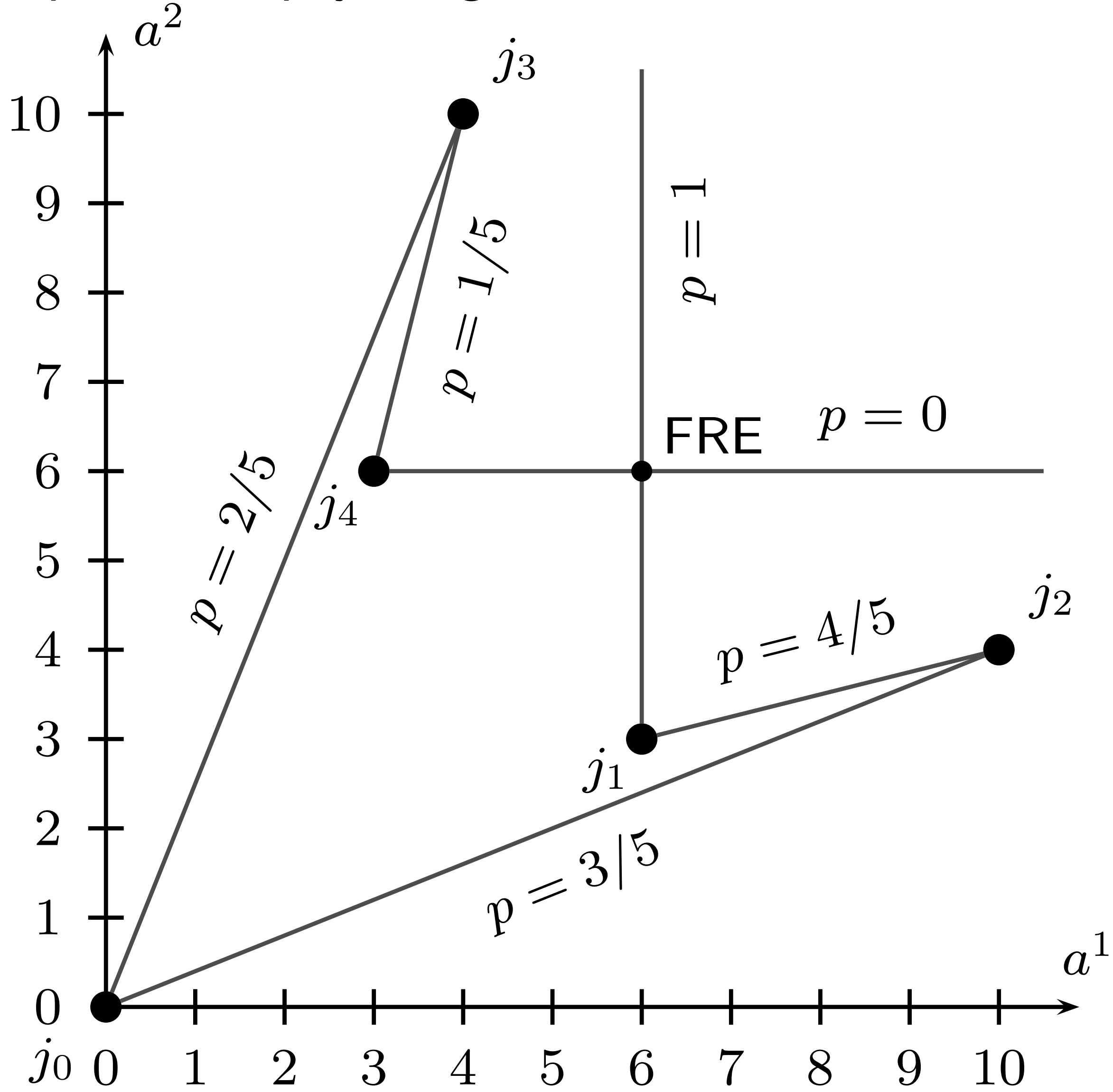

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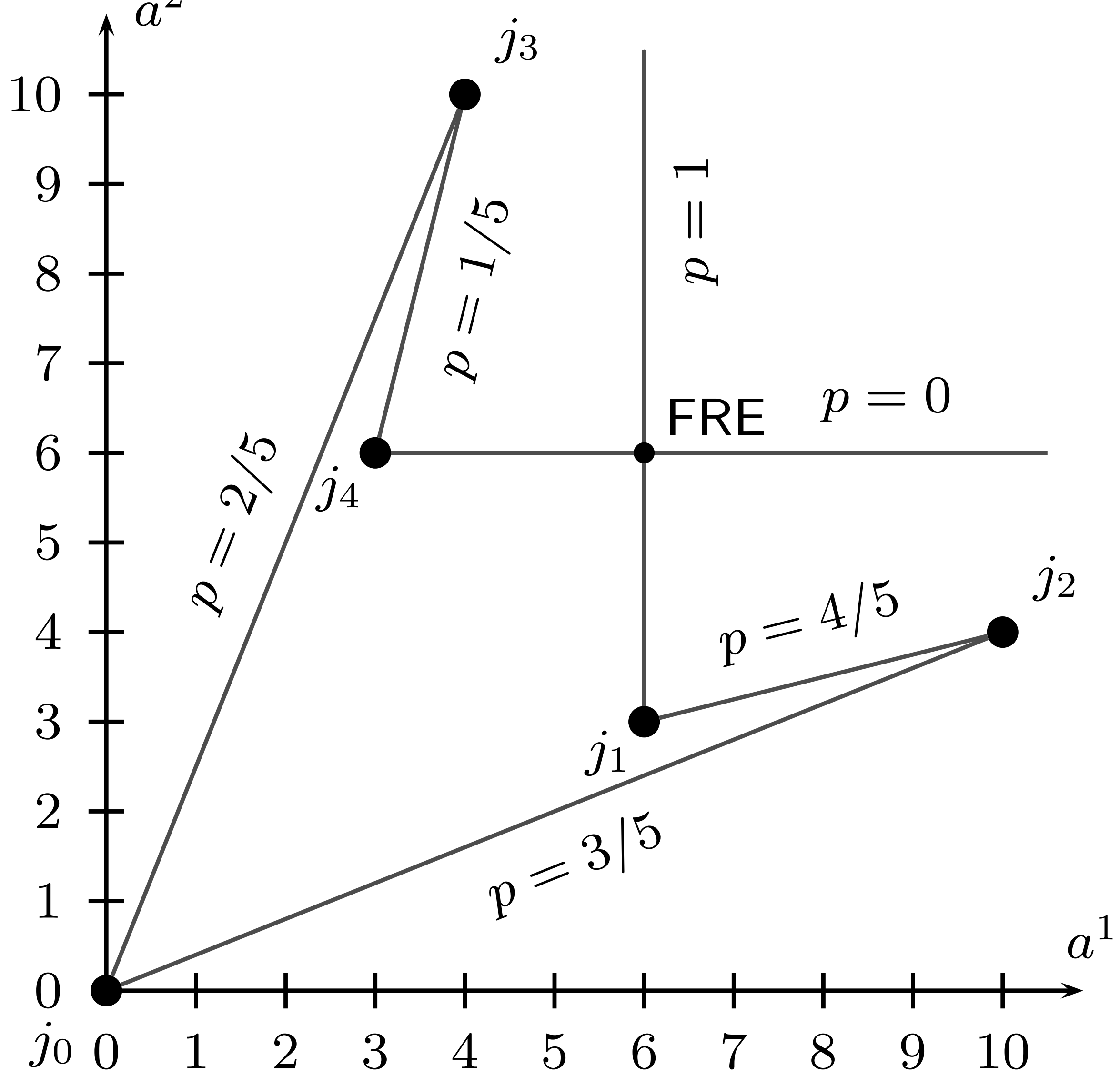

From the equilibrium characterization theorem for $\Gamma^0_S(p)$, there is only two types of equilibria in the single-stage cheap talk game: NRE and FRE

But in the 3-stage cheap talk game, when $p = 3/10$, the interim payoff $(3, 6)$ can be obtained as follows, where $y = (2/5)j_0 + (3/5)j_3$

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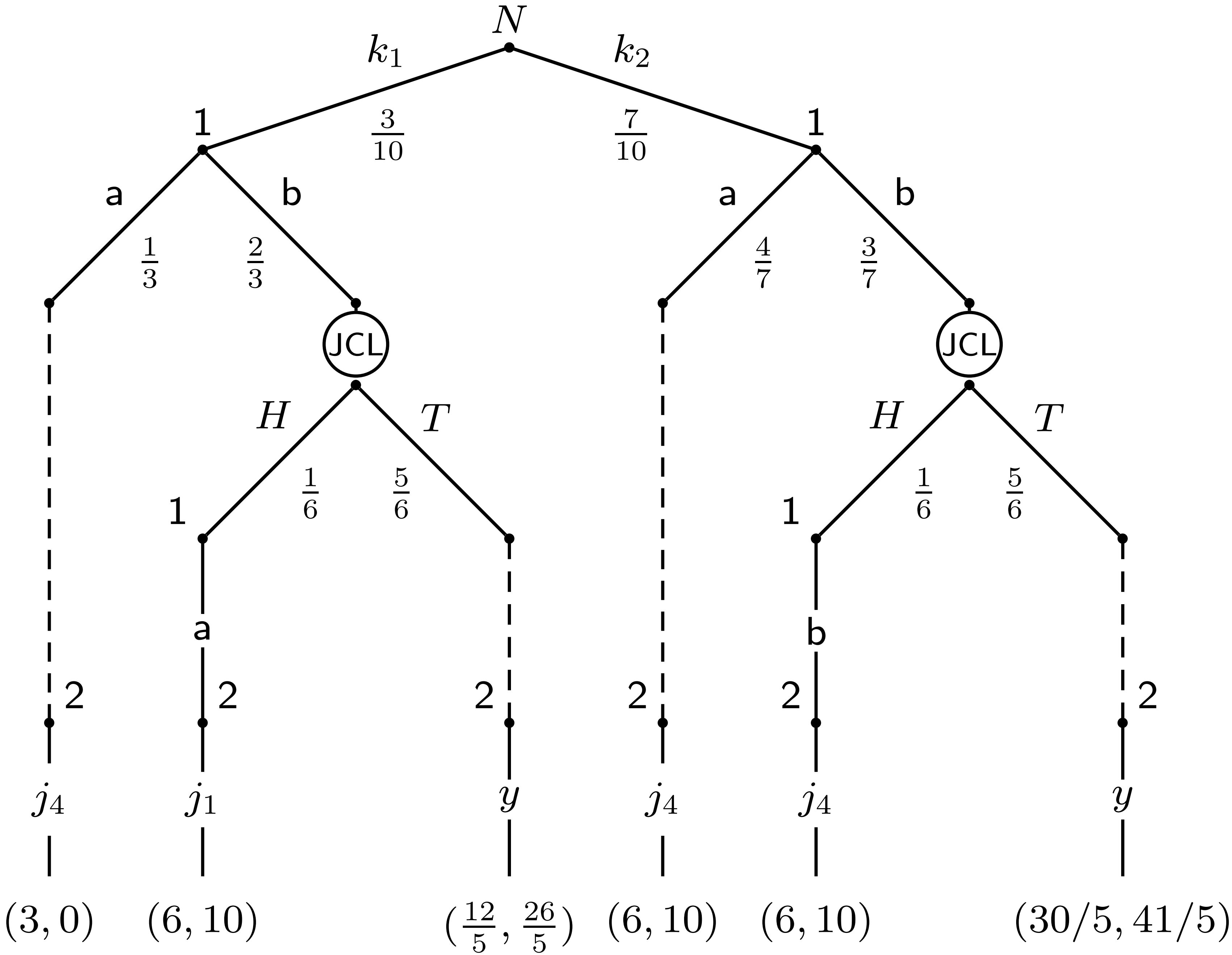

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Adding a signalling stage before the JCL allows a second convexification at p fixed

Hence, for all $p \in [1/5, 2/5]$ (in particular, $p = 3/10$) we get $[j_3, j_4, \mathsf{FRE}]$ (in particular, $a = (3, 6)$) with three communication stages

A subset of $\mathbb{R}^2 \times \mathbb{R} \times [0,1]$ is diconvex if it is convex in (β, p) when a is fixed, and convex in (a, β) when p is fixed. di-co$\,(E)$ is the smallest diconvex set containing E

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Theorem. (Hart, 1985, Forges, 1994, Aumann and Hart, 2003) Let $p \in (0,1)$. A payoff (a, β) is an equilibrium payoff of some bilateral communication game $\Gamma_n^0(p)$, for some length n, if and only if (a, β, p) belongs to di-co$\,(\mathrm{gr}\,\mathcal{E}^+)$, the set of all points obtained by diconvexifying the set $\mathrm{gr}\,\mathcal{E}^+$

1.4 Communication with No Deadline

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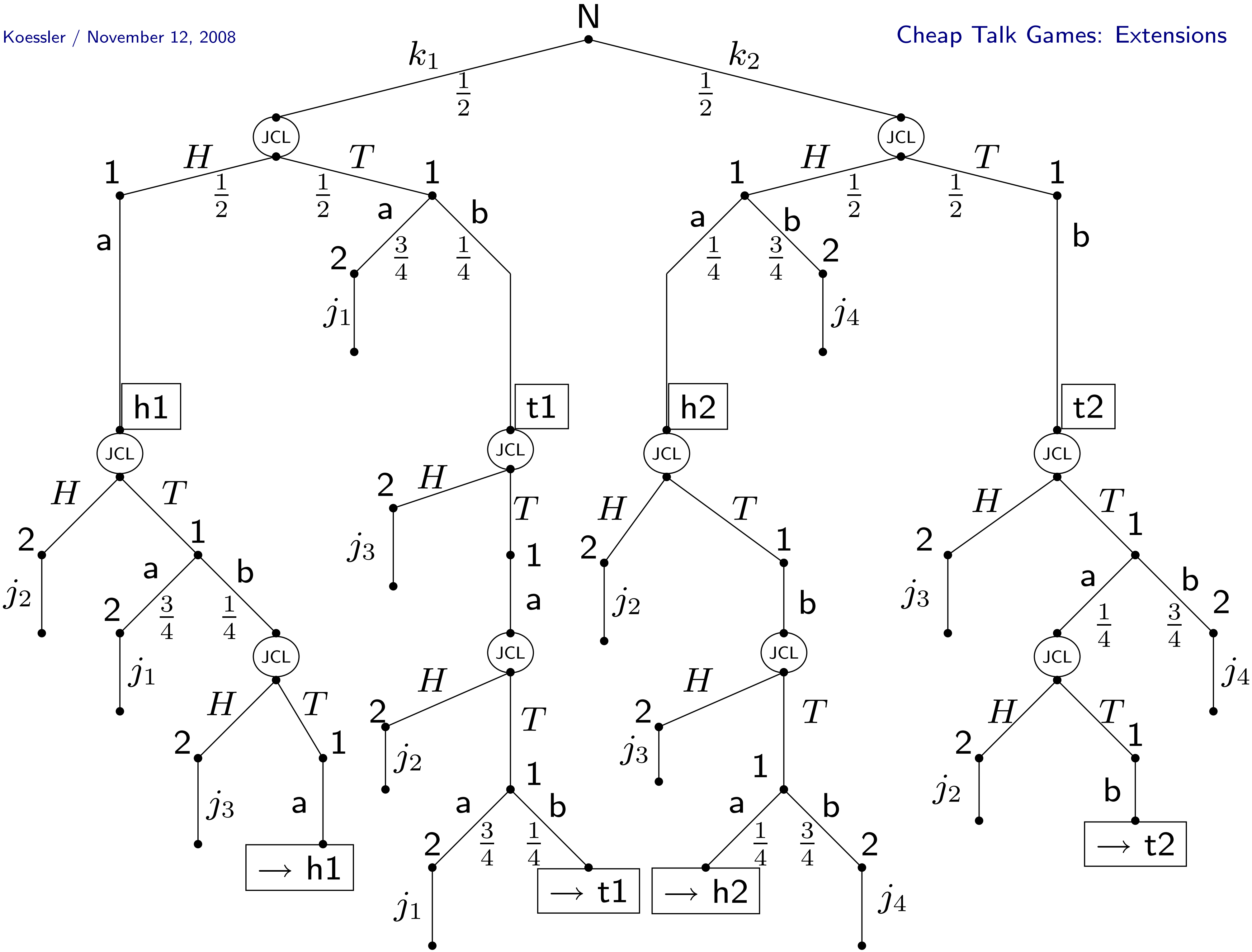
When the number of communication stages, n, is not fixed in advance, the job candidate can even achieve the expected payoff $(7, 7)$ when $p = 1/2$


N
k_1
$\frac{1}{2}$
k_2
$\frac{1}{2}$
JCL
H
T
1
a
b
2
$\frac{3}{4}$
$\frac{1}{4}$
j_1
j_2
j_3
j_4
h1
t1
h2
t2
→ h1
→ t1
→ h2
→ t2

1.5 Conversation in Crawford and Sobel's Model

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Configuration 1: Intermediate Bias ($b = 1/10$).

When $b = 1/10$, there is two possible types of equilibria in the model of Crawford and Sobel: a NRE and a 2-partitional equilibrium

The 2-partitional equilibrium is the most efficient one, and is given by

$$\sigma_1(t) = \begin{cases} m_1 & \text{if } t \in [0, x) \\ m_2 & \text{if } t \in [x, 1], \end{cases}$$

where $x = 1/2 - (2/10)(2-1) = 3/10$, $\sigma_2(m_1) = x/2 = 3/20$, $\sigma_2(m_2) = (1+x)/2 = 13/20$

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and

$$EU_2 = -\frac{1}{12 \times 2^2} - \frac{(1/10)^2(2^2-1)}{3} = -37/1200$$

$$EU_1 = EU_2 - b^2 = -49/1200$$

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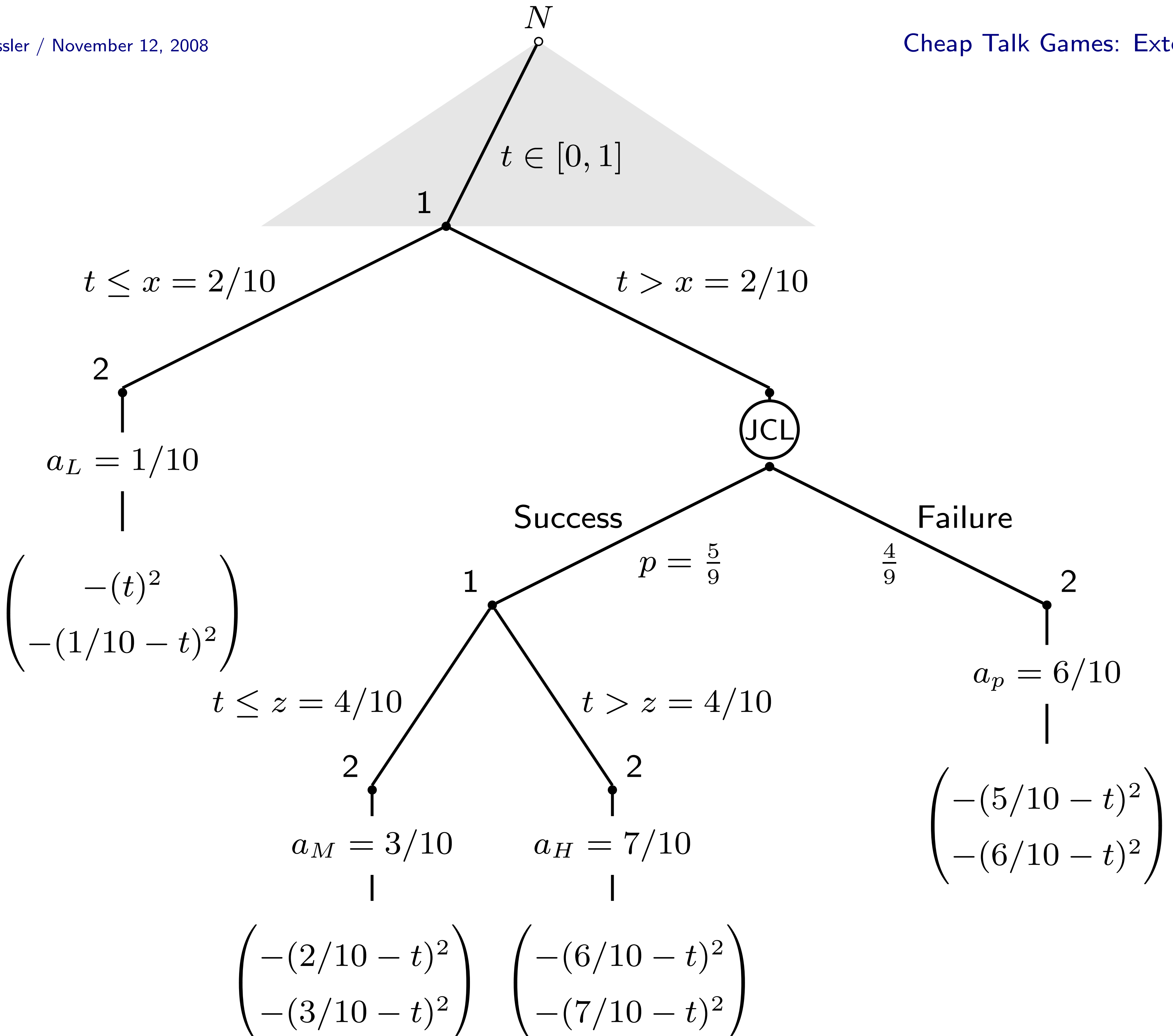
The following equilibrium in the 3-stage game is (ex-ante) Pareto improving


N
$t \in [0,1]$
1
$t \leq x = 2/10$
$t > x = 2/10$
2
JCL
$a_L = 1/10$
Success
Failure
$p = \frac{5}{9}$
$\frac{4}{9}$
1
2
$-(t)^2$
$-(1/10 - t)^2$
$t \leq z = 4/10$
$t > z = 4/10$
$a_p = 6/10$
2
2
$-(5/10 - t)^2$
$-(6/10 - t)^2$
$a_M = 3/10$
$a_H = 7/10$
$-(2/10 - t)^2$
$-(3/10 - t)^2$
$-(6/10 - t)^2$
$-(7/10 - t)^2$

Ex-ante expected payoffs:

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$$EU_1 = -\int_0^{2/10} t^2\,dt - \frac{5}{9}\left[\int_{2/10}^{4/10}(2/10-t)^2\,dt + \int_{4/10}^{1}(6/10-t)^2\,dt\right]$$
$$-\frac{4}{9}\int_{2/10}^{1}(5/10-t)^2\,dt = -\frac{48}{1200}$$
$$EU_2 = EU_1 + b^2 = -\frac{36}{1200}$$

Configuration 2: High Bias ($b = 7/24$).

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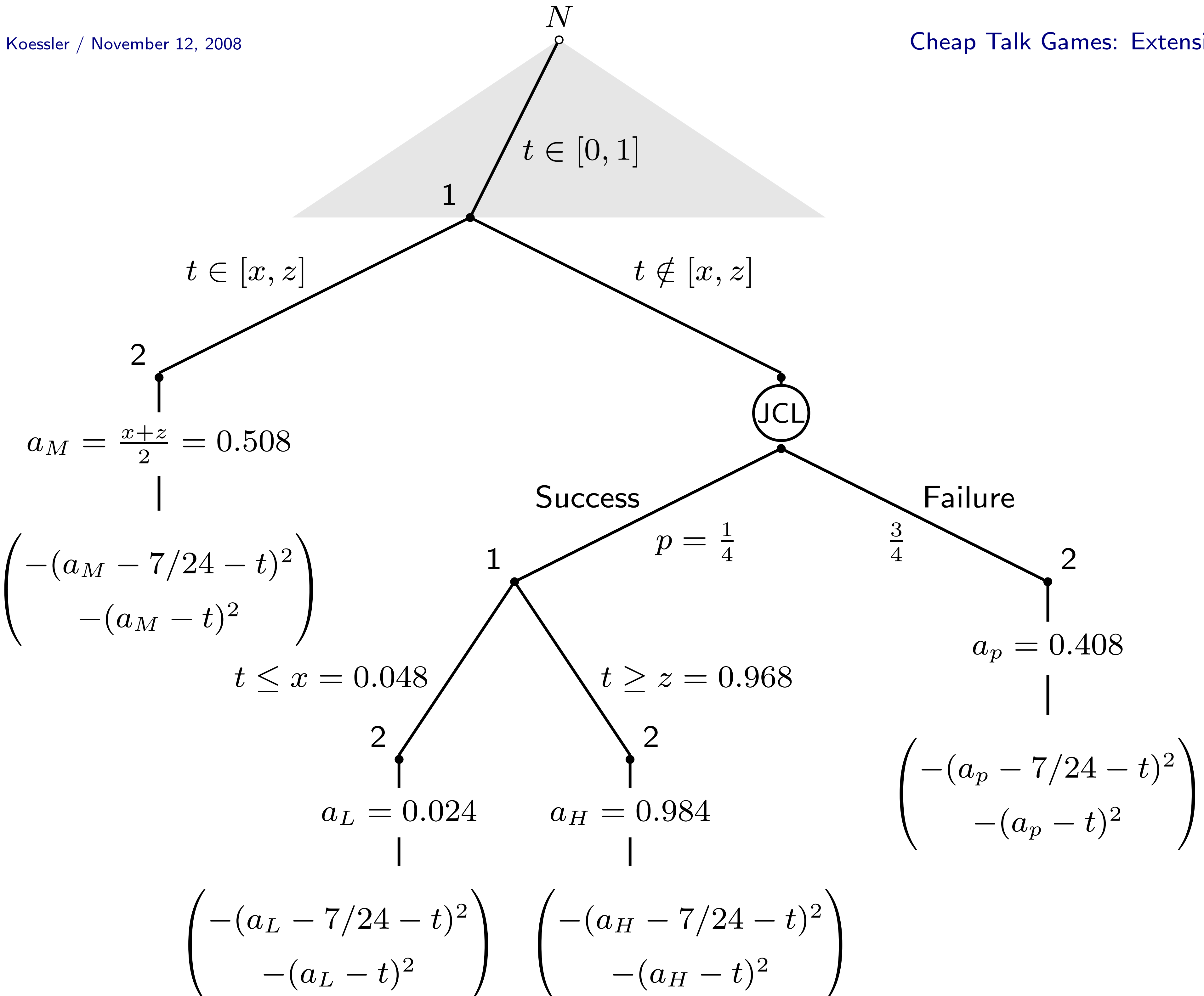
When $b = 7/24 > 1/4$ the unique equilibrium with unilateral communication in NR

The following (non-monotonic) equilibrium of the 3-stage game, where $x = 0.048$ and $z = 0.968$, Pareto dominates this NRE


N
$t \in [0,1]$
1
$t \in [x,z]$
$t \notin [x,z]$
2
JCL
$a_M = \frac{x+z}{2} = 0.508$
$\begin{pmatrix} -(a_M - 7/24 - t)^2 \\ -(a_M - t)^2 \end{pmatrix}$
Success
Failure
$p = \frac{1}{4}$
$\frac{3}{4}$
1
2
$a_p = 0.408$
$t \leq x = 0.048$
$t \geq z = 0.968$
2
2
$\begin{pmatrix} -(a_p - 7/24 - t)^2 \\ -(a_p - t)^2 \end{pmatrix}$
$a_L = 0.024$
$a_H = 0.984$
$\begin{pmatrix} -(a_L - 7/24 - t)^2 \\ -(a_L - t)^2 \end{pmatrix}$
$\begin{pmatrix} -(a_H - 7/24 - t)^2 \\ -(a_H - t)^2 \end{pmatrix}$

More generally, Krishna and Morgan (2004) show that

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- for all $b < 1/8$, there is a monotonic Nash equilibrium outcome of the 3-stage communication game which Pareto dominates all equilibrium outcomes of the unilateral communication game (Krishna and Morgan, 2004, Theorem 1)

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- for all $b < 1/8$, there is a monotonic Nash equilibrium outcome of the 3-stage communication game which Pareto dominates all equilibrium outcomes of the unilateral communication game (Krishna and Morgan, 2004, Theorem 1)
- for all $b \in (1/8, 1/\sqrt{8})$, there is a non-monotonic Nash equilibrium outcome of the 3-stage communication game which Pareto dominates the unique NR equilibrium outcome of the unilateral communication game (Krishna and Morgan, 2004, Theorem 2)

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- for all $b \in (1/8, 1/\sqrt{8})$, there is a non-monotonic Nash equilibrium outcome of the 3-stage communication game which Pareto dominates the unique NR equilibrium outcome of the unilateral communication game (Krishna and Morgan, 2004, Theorem 2)
- for all $b > 1/8$ it is not possible to Pareto improve the unique NR equilibrium outcome of the unilateral communication game with monotonic equilibria Krishna and Morgan (2004, Proposition 3)

2 Mediated Communication: Correlated and Communication Equilibria

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2.1 Complete Information Games: Correlated Equilibrium

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At least, players are able to achieve the convex hull of the set of Nash equilibrium payoffs, by using jointly controlled lotteries, or simply by letting a mediator publicly reveal the realization of a random device

For example, tossing a fair coin allows to achieve the outcome $\mu = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$

with payoffs $(\frac{9}{2}, \frac{9}{2})$ in the chicken game:

	a	b
a	$(2,7)$	$(6,6)$
b	$(0,0)$	$(7,2)$

More generally, adding any system for preplay communication generates some information system

$$\langle \Omega,\, p,\, (\mathcal{P}_i)_{i \in N} \rangle$$

so a Nash equilibrium of this extended game exactly corresponds to

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Definition (Aumann, 1974) A **correlated equilibrium** (CE) of the normal form game

$$\langle N,\, (A_i)_{i \in N},\, (u_i)_{i \in N} \rangle$$

is a pure strategy Nash equilibrium of the Bayesian game

$$\langle N,\, \Omega,\, p,\, (\mathcal{P}_i)_i,\, (A_i)_i,\, (u_i)_i \rangle$$

where $u_i(a;\omega) = u_i(a)$, i.e., a profile of pure strategies $s = (s_1, \ldots, s_n)$ such that for every $i \in N$ and every strategy r_i of player i:

$$\sum_{\omega \in \Omega} p(\omega)\; u_i(s_i(\omega), s_{-i}(\omega)) \geq \sum_{\omega \in \Omega} p(\omega)\; u_i(r_i(\omega), s_{-i}(\omega))$$

- Correlated equilibrium outcome $\mu \in \Delta(A)$, where

$$\mu(a) = p(\{\omega \in \Omega : s(\omega) = a\})$$

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- Correlated equilibrium payoff $\sum_{a \in A} \mu(a) u_i(a)$, $i = 1, \ldots, n$

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$$\mathcal{P}_1 = \{\underbrace{\{\omega_1, \omega_2\}}_{a}, \underbrace{\{\omega_3\}}_{b}\}$$

$$\mathcal{P}_2 = \{\underbrace{\{\omega_1\}}_{a}, \underbrace{\{\omega_2, \omega_3\}}_{b}\}$$

	a	b
a	$(2,7)$	$(6,6)$
b	$(0,0)$	$(7,2)$

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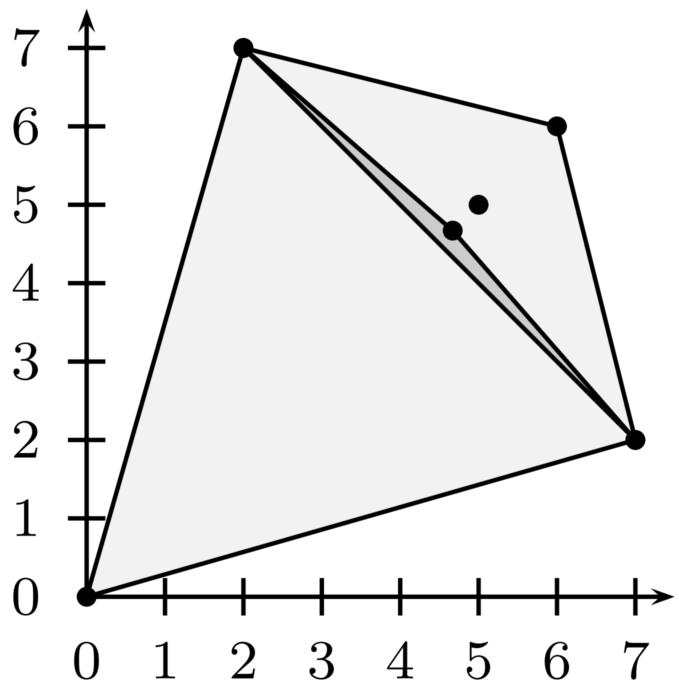
➡ Correlated equilibrium payoff $(5, 5) \notin \mathrm{co}\{\text{EN}\}$



A correlated equilibrium can Pareto dominate every Nash equilibrium

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The following game, where $z^+ = z + \varepsilon$ and $z^- = z - \varepsilon$

$$\begin{pmatrix} 0,0 & x^+, y^- & x^-, y^+ \\ x^-, y^+ & 0,0 & x^+, y^- \\ x^+, y^- & x^-, y^+ & 0,0 \end{pmatrix}$$

has a unique Nash equilibrium

$$\begin{pmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{pmatrix} \text{ with payoffs } (\frac{2}{3}x, \frac{2}{3}y)$$

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while there is a correlated equilibrium

$$\begin{pmatrix} 0 & 1/6 & 1/6 \\ 1/6 & 0 & 1/6 \\ 1/6 & 1/6 & 0 \end{pmatrix} \text{ with payoffs } (x, y)$$

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Proposition 1 *Every correlated equilibrium outcome of a normal form game* $\langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$ *is a* **canonical correlated equilibrium** *outcome, where the information structure and strategies are given by:*

- $\Omega = A$
- $\mathcal{P}_i = \{\{a \in A : a_i = b_i\} : b_i \in A_i\}$ *for every* $i \in N$
- $s_i(a) = a_i$ *for every* $a \in A$ *and* $i \in N$

2.2 Incomplete Information Games: Communication Equilibrium

A **communication equilibrium** of a Bayesian game is a Nash equilibrium of some preplay and interim communication extension of the game

- The communication system should possibly include a mediator who can send outputs but also receive **inputs** from the players (two-way communication)
- A communication equilibrium outcome is a mapping $\mu : T \to \Delta(A)$

A **canonical communication equilibrium** of a Bayesian game is a Nash equilibrium of the one-stage communication extension of the game in which each player

- first, truthfully reveals his type to the mediator
- then, follows the recommendation of action of the mediator

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i.e. for all $i \in N$, $t_i \in T_i$, $s_i \in T_i$ and $\delta : A_i \to A_i$,

$$\sum_{t_{-i} \in T_{-i}} \sum_{a \in A} p(t_{-i} \mid t_i) \mu(a \mid t) u_i(a, t) \geq$$

$$\sum_{t_{-i} \in T_{-i}} \sum_{a \in A} p(t_{-i} \mid t_i) \mu(a \mid t_{-i}, s_i) u_i(a_{-i}, \delta(a_i), t)$$

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Revelation Principle for Bayesian Games: The set of communication equilibrium outcomes coincides with the set of canonical communication equilibrium outcomes

Example. The geometric characterization theorem shows that face-to-face (even multistage) communication cannot matter in the following game:

	j_1	j_2	j_3
k_1	$3, 3$	$1, 2$	$0, 0$
k_2	$2, 0$	$3, 2$	$1, 3$

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But mediated or noisy communication allows some (Pareto improving) information transmission

For example, when $\Pr(k_1) = 1/2$

$$\mu(k_1) = \frac{1}{2} j_1 + \frac{1}{2} j_2 \quad \text{and} \quad \mu(k_2) = j2$$

is a Pareto improving communication equilibrium

Mediation in the (quadratic) model of Crawford and Sobel (1982)

Goltsman et al. (2007): (Face-to-face) cheap talk is as efficient as mediated communication if and only if the bias b does not exceed $1/8$

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