Strategic Information Transmission: Persuasion Games

Strategic Information Transmission: Persuasion Games

Outline

(November 22, 2007)

- The revelation principle revisited
- Hard evidence and information certification in games
- Geometric Characterization of Nash Equilibrium Outcomes
- Sceptical strategies and worst case inferences in monotonic relationships
- Persuasion with type-dependent biases (Seidmann and Winter, 1997)
- Long persuasion games

Verifiable Information and Certification

Verifiable Information and Certification

Some private information like

- individual preferences
- tastes
- ideas
- intentions
- the quality of a project
- the cost of effort

are usually non-certifiable / non-provable, and cannot be objectively measured by a third party

Verifiable Information and Certification

Some private information like

- individual preferences
- tastes
- ideas
- intentions
- the quality of a project
- the cost of effort

are usually non-certifiable / non-provable, and cannot be objectively measured by a third party

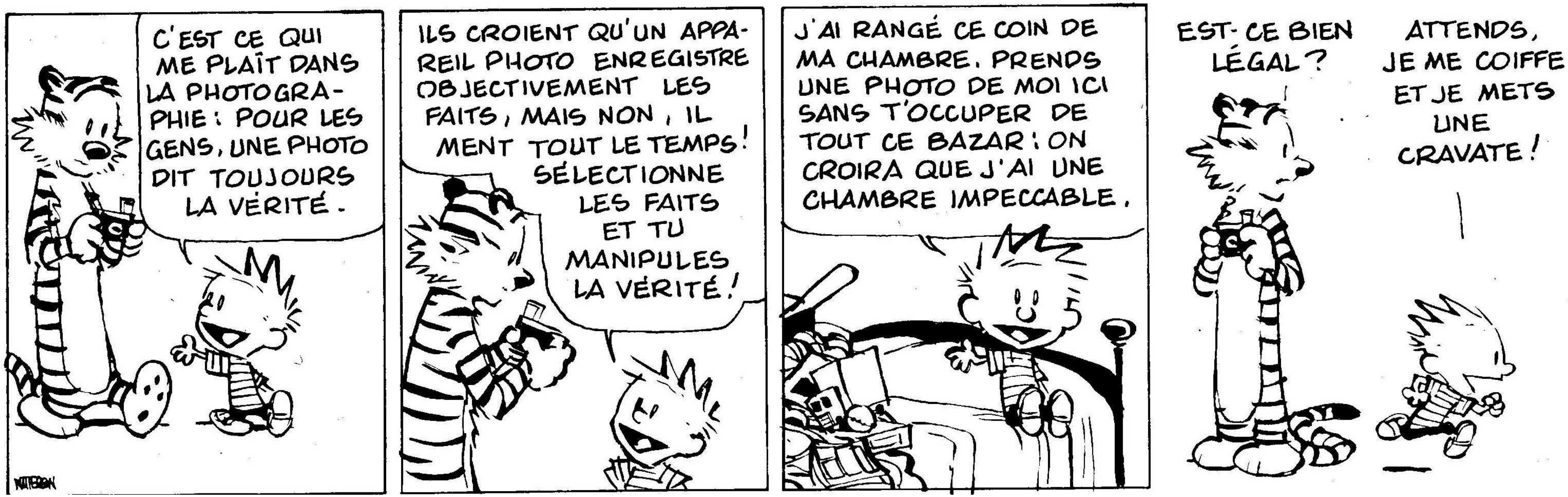
On the other hand,

- the health or income of an individual
- the debt of a firm
- the history of a car maintenance
- a doctor's degree

may be directly certified, or authenticated by a third party

How does one person make another believe something ? The answer depends importantly on the factual question, "Is it true ?" It is easier to prove the truth of something that is true than of something false. To prove the truth about our health we can call on a reputable doctor ; to prove the truth about our costs or income we may let the person look at books that have been audited by a reputable firm or the Bureau of Internal Revenue. But to persuade him of something false we may have no such convincing evidence.

Schelling, 1960, p. 23.


C'EST CE QUI ME PLAÎT DANS LA PHOTOGRAPHIE : POUR LES GENS, UNE PHOTO DIT TOUJOURS LA VÉRITÉ.
ILS CROIENT QU'UN APPAREIL PHOTO ENREGISTRE OBJECTIVEMENT LES FAITS, MAIS NON, IL MENT TOUT LE TEMPS !
SÉLECTIONNE LES FAITS ET TU MANIPULES LA VÉRITÉ !
J'AI RANGÉ CE COIN DE MA CHAMBRE. PRENDS UNE PHOTO DE MOI ICI SANS T'OCCUPER DE TOUT CE BAZAR : ON CROIRA QUE J'AI UNE CHAMBRE IMPECCABLE.
EST-CE BIEN LÉGAL ?
ATTENDS, JE ME COIFFE ET JE METS UNE CRAVATE !

The information that **can** be revealed by a player may depend on his actual state of knowledge $\Rightarrow M_i(k)$: set of messages of player i when his type is k

The information that **can** be revealed by a player may depend on his actual state of knowledge $\Rightarrow$ $\boldsymbol{M_i(k)}$: set of messages of player $\boldsymbol{i}$ when his type is $\boldsymbol{k}$

☞ Physical proofs ("hard information")

- Documents
- Observable characteristics of a product
- Endowments, costs
- Income tax return
- Claims about health conditions

The information that **can** be revealed by a player may depend on his actual state of knowledge $\Rightarrow$ $\boldsymbol{M_i(k)}$: set of messages of player $\boldsymbol{i}$ when his type is $\boldsymbol{k}$

☞ Physical proofs ("hard information")

- Documents
- Observable characteristics of a product
- Endowments, costs
- Income tax return
- Claims about health conditions

☞ Legal constraints

- Revelation of accounting data
- Advertisement, labels, guarantee of quality, ...

The information that **can** be revealed by a player may depend on his actual state of knowledge $\Rightarrow$ $\boldsymbol{M_i(k)}$: set of messages of player $\boldsymbol{i}$ when his type is $\boldsymbol{k}$

☞ Physical proofs ("hard information")

- Documents
- Observable characteristics of a product
- Endowments, costs
- Income tax return
- Claims about health conditions

☞ Legal constraints

- Revelation of accounting data
- Advertisement, labels, guarantee of quality, . . .

☞ Psychological constraints

- Honesty / Observable emotions (blushing, stress . . .)

The Revelation Principle Revisited

The Revelation Principle Revisited

Set of possible announcements for an agent of type θ:
$M(\theta) \subseteq \Theta$, with $\theta \in M(\theta)$

The Revelation Principle Revisited

Set of possible announcements for an agent of type $\boldsymbol{\theta}$:
$\boldsymbol{M(\theta) \subseteq \Theta}$, with $\boldsymbol{\theta \in M(\theta)}$

How an optimal mechanism and the revelation principle is affected by this new feature?

➥ Green and Laffont (1986)

The Revelation Principle Revisited

Set of possible announcements for an agent of type $\boldsymbol{\theta}$:
$\boldsymbol{M(\theta) \subseteq \Theta}$, with $\boldsymbol{\theta \in M(\theta)}$

How an optimal mechanism and the revelation principle is affected by this new feature?

➥ Green and Laffont (1986)

Utility of the agent when his type is $\boldsymbol{\theta}$ and the decision is $\boldsymbol{x \in X}$:

$$\boldsymbol{u(x, \theta)}$$

The Revelation Principle Revisited

Set of possible announcements for an agent of type θ:
$M(\theta) \subseteq \Theta$, with $\theta \in M(\theta)$

How an optimal mechanism and the revelation principle is affected by this new feature?

➥ Green and Laffont (1986)

Utility of the agent when his type is θ and the decision is $x \in X$:

$$u(x, \theta)$$

Direct mechanism:

$$x : \Theta \longrightarrow X$$

(More generally, a mechanism is $x : \mathcal{M} \longrightarrow X$, where $\mathcal{M}$ is any set of messages)

An **allocation**, or **social choice function** $y : \Theta \longrightarrow X$ is **directly M-implementable** if there exists a direct mechanism $x : \Theta \longrightarrow X$ such that

$$x(m^*(\theta)) = y(\theta)$$

where m^* is the optimal reporting strategy of the agent, i.e.,

$$m^*(\theta) \in \arg \max_{m \in M(\theta)} u(x(m), \theta)$$

An **allocation**, or **social choice function** $y : \Theta \longrightarrow X$ is **directly M-implementable** if there exists a direct mechanism $x : \Theta \longrightarrow X$ such that

$$x(m^*(\theta)) = y(\theta)$$

where m^* is the optimal reporting strategy of the agent, i.e.,

$$m^*(\theta) \in \arg \max_{m \in M(\theta)} u(x(m), \theta)$$

An allocation $y : \Theta \longrightarrow X$ is directly and **truthfully** M-implementable if there exists a direct mechanism $x : \Theta \longrightarrow X$ such that

$$x(m^*(\theta)) = y(\theta)$$

and $m^*(\theta) = \theta \in \arg\max_{m \in M(\theta)} u(y(m), \theta)$ for all $\theta \in \Theta$ (standard informational incentive constraint)

Standard setting (non-verifiable types): $M(\theta) = \Theta$ for all $\theta \in \Theta$, and the revelation principle applies: an allocation is implementable if and only if it is directly and truthfully implementable

Standard setting (non-verifiable types): $M(\theta) = \Theta$ for all $\theta \in \Theta$, and the revelation principle applies: an allocation is implementable if and only if it is directly and truthfully implementable

$$\begin{tikzcd} \Theta \arrow[r, "m^*(\cdot)"] \arrow[rr, bend right, "y(\cdot) = x \circ m^*(\cdot)"'] & \mathcal{M} \arrow[r, "x(\cdot)"] & X \end{tikzcd}$$

Clearly, y generates the same allocation as x, and truthful revelation $m(\theta) = \theta$ is optimal for the agent with the new mechanism

The revelation principle does not apply, in general, with partially verifiable types

The revelation principle does not apply, in general, with partially verifiable types

Example 1 (Failure of the revelation principle) $\boldsymbol{\Theta = \{\theta_1, \theta_2, \theta_3\}}$, $\boldsymbol{X = \{x_1, x_2, x_3\}}$,

$$\begin{aligned} M(\theta_1) &= \{\theta_1, \theta_2\} \\ M(\theta_2) &= \{\theta_2, \theta_3\} \\ M(\theta_3) &= \{\theta_3\} \end{aligned}$$

$$u = \begin{array}{cccc} & x_1 & x_2 & x_3 \\ \theta_1 & \boxed{0} & 1 & 2 \\ \theta_2 & 1 & \boxed{2} & 0 \\ \theta_3 & 0 & \boxed{1} & 2 \end{array}$$

and $y(\theta_1) = x_1$, $y(\theta_2) = y(\theta_3) = x_2$

Clearly, y is not truthfully implementable (θ_1 claims to be $m^*(\theta_1) = \theta_2$)

Nevertheless, y can be implemented with the mechanism

$$x(\theta_1) = x(\theta_2) = x_1$$
$$x(\theta_3) = x_2$$

Nevertheless, y can be implemented with the mechanism

$$x(\theta_1) = x(\theta_2) = x_1$$
$$x(\theta_3) = x_2$$

In this case, the optimal strategy of the agent in not truthful:

$$\begin{aligned} m^*(\theta_1) = &\ \{\theta_1, \theta_2\} \\ m^*(\theta_2) = &\ \theta_3 \\ m^*(\theta_3) = &\ \theta_3 \end{aligned}$$

Nevertheless, $\boldsymbol{y}$ can be implemented with the mechanism

$$\begin{aligned} x(\theta_1) = x(\theta_2) &= x_1 \\ x(\theta_3) &= x_2 \end{aligned}$$

In this case, the optimal strategy of the agent in not truthful:

$$\begin{aligned} m^*(\theta_1) &= \{\theta_1, \theta_2\} \\ m^*(\theta_2) &= \theta_3 \\ m^*(\theta_3) &= \theta_3 \end{aligned}$$

but $\boldsymbol{y}$ is implemented:

$$\begin{aligned} x \circ m^*(\theta_1) &= x_1 = y(\theta_1) \\ x \circ m^*(\theta_2) &= x_2 = y(\theta_2) \\ x \circ m^*(\theta_3) &= x_2 = y(\theta_3) \end{aligned}$$

Nested Range Condition

Nested Range Condition

The message correspondence M satisfied the **Nested Range Condition (NRC)** if for all θ, $\theta' \in \Theta$, we have

$$\theta' \in M(\theta) \;\Rightarrow\; M(\theta') \subseteq M(\theta)$$

Nested Range Condition

The message correspondence M satisfied the **Nested Range Condition (NRC)** if for all θ, $\theta' \in \Theta$, we have

$$\theta' \in M(\theta) \ \Rightarrow \ M(\theta') \subseteq M(\theta)$$

This condition is not satisfied in the previous example because $\theta_2 \in M(\theta_1)$ but $M(\theta_2) = \{\theta_2, \theta_3\} \nsubseteq M(\theta_1) = \{\theta_1, \theta_2\}$

Nested Range Condition

The message correspondence M satisfied the **Nested Range Condition (NRC)** if for all θ, $\theta' \in \Theta$, we have

$$\theta' \in M(\theta) \;\Rightarrow\; M(\theta') \subseteq M(\theta)$$

This condition is not satisfied in the previous example because $\theta_2 \in M(\theta_1)$ but $M(\theta_2) = \{\theta_2, \theta_3\} \nsubseteq M(\theta_1) = \{\theta_1, \theta_2\}$

Example where NRC is satisfied: unidirectional distortions. Letting Θ be ordered by $\preceq$, $M(\theta) = \{\tilde{\theta} \in \Theta : \tilde{\theta} \preceq \theta\}$ satisfies NRC

Application: claims about income or health that cannot be imitated by lower types

Proposition 1 (Green and Laffont, 1986) *If M satisfies the Nested Range Condition then the revelation principle applies: for every decision set X and utility function $u : X \times \Theta \longrightarrow \mathbb{R}$, the set of directly M-implementable allocations coincides with the set of directly and truthfully M-implementable allocations*

Proof. Consider a mechanism $\boldsymbol{x}$ that implements allocation $\boldsymbol{y}$, but assume that $\boldsymbol{y}$ is not truthfully implementable. We show that NRC is not satisfied

Proof. Consider a mechanism x that implements allocation y, but assume that y is not truthfully implementable. We show that NRC is not satisfied

Since y is not truthfully implementable, there exist θ_1 and θ_2 such that $\theta_2 \in M(\theta_1)$ and

$$u(y(\theta_2), \theta_1) > u(y(\theta_1), \theta_1)$$

Proof. Consider a mechanism x that implements allocation y, but assume that y is not truthfully implementable. We show that NRC is not satisfied

Since y is not truthfully implementable, there exist θ_1 and θ_2 such that $\theta_2 \in M(\theta_1)$ and

$$u(y(\theta_2), \theta_1) > u(y(\theta_1), \theta_1)$$

Since x implements y we have

- $x(\theta) \neq y(\theta_2)$ for all $\theta \in M(\theta_1)$ (otherwise, θ_1 deviates)
- $x(m^*(\theta_2)) = y(\theta_2)$, where $m^*(\theta_2) \in M(\theta_2)$

Proof. Consider a mechanism $\boldsymbol{x}$ that implements allocation $\boldsymbol{y}$, but assume that $\boldsymbol{y}$ is not truthfully implementable. We show that NRC is not satisfied

Since $\boldsymbol{y}$ is not truthfully implementable, there exist $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_2}$ such that $\boldsymbol{\theta_2 \in M(\theta_1)}$ and

$$\boldsymbol{u(y(\theta_2), \theta_1) > u(y(\theta_1), \theta_1)}$$

Since $\boldsymbol{x}$ implements $\boldsymbol{y}$ we have

- $\boldsymbol{x(\theta) \neq y(\theta_2)}$ for all $\boldsymbol{\theta \in M(\theta_1)}$ (otherwise, $\boldsymbol{\theta_1}$ deviates)
- $\boldsymbol{x(m^*(\theta_2)) = y(\theta_2)}$, where $\boldsymbol{m^*(\theta_2) \in M(\theta_2)}$

Hence:

$$\begin{aligned} \boldsymbol{\theta_2} &\in \boldsymbol{M(\theta_1)} \\ \boldsymbol{m^*(\theta_2)} &\in \boldsymbol{M(\theta_2)} \quad \Rightarrow \quad \boldsymbol{M(\theta_2) \not\subseteq M(\theta_1)} \\ \boldsymbol{m^*(\theta_2)} &\notin \boldsymbol{M(\theta_1)} \end{aligned}$$

which violates NRC

General Mechanisms (not necessarily direct, with no restriction on communication)

$$x : \mathcal{M} \to X$$

where $\mathcal{M}$ is any message set (not necessarily Θ)

General Mechanisms (not necessarily direct, with no restriction on communication)

$$x : \mathcal{M} \to X$$

where $\mathcal{M}$ is any message set (not necessarily Θ)

Example 2 (Failure of the revelation principle 2) Consider Example 2 with another allocation $y(\theta_i) = x_i$

$$\begin{aligned} M(\theta_1) &= \{\theta_1, \theta_2\} \\ M(\theta_2) &= \{\theta_2, \theta_3\} \\ M(\theta_3) &= \{\theta_3\} \end{aligned} \qquad u = \begin{array}{cccc} & x_1 & x_2 & x_3 \\ \theta_1 & \boxed{0} & 1 & 2 \\ \theta_2 & 1 & \boxed{2} & 0 \\ \theta_3 & 0 & 1 & \boxed{2} \end{array}$$

Clearly, y is not directly implementable (truthfully or not)

General Mechanisms (not necessarily direct, with no restriction on communication)

$$x : \mathcal{M} \to X$$

where $\mathcal{M}$ is any message set (not necessarily Θ)

Example 2 (Failure of the revelation principle 2) Consider Example 2 with another allocation $y(\theta_i) = x_i$

$$\begin{aligned} M(\theta_1) &= \{\theta_1, \theta_2\} \\ M(\theta_2) &= \{\theta_2, \theta_3\} \\ M(\theta_3) &= \{\theta_3\} \end{aligned} \qquad u = \begin{array}{cccc} & x_1 & x_2 & x_3 \\ \theta_1 & \boxed{0} & 1 & 2 \\ \theta_2 & 1 & \boxed{2} & 0 \\ \theta_3 & 0 & 1 & \boxed{2} \end{array}$$

Clearly, y is not directly implementable (truthfully or not)

However, it can be implemented by asking the agent to send **two** messages

$$\begin{aligned} M(\theta_1) &= \{\theta_1, \theta_2\} \\ M(\theta_2) &= \{\theta_2, \theta_3\} \\ M(\theta_3) &= \{\theta_3\} \end{aligned}$$

$$u = \begin{array}{cccc} & x_1 & x_2 & x_3 \\ \theta_1 & \boxed{0} & 1 & 2 \\ \theta_2 & 1 & \boxed{2} & 0 \\ \theta_3 & 0 & 1 & \boxed{2} \end{array}$$

$$
\begin{aligned}
M(\theta_1) &= \{\theta_1, \theta_2\} \\
M(\theta_2) &= \{\theta_2, \theta_3\} \\
M(\theta_3) &= \{\theta_3\}
\end{aligned}
\qquad
u = \begin{array}{cccc}
 & x_1 & x_2 & x_3 \\
\theta_1 & \boxed{0} & 1 & 2 \\
\theta_2 & 1 & \boxed{2} & 0 \\
\theta_3 & 0 & 1 & \boxed{2}
\end{array}
$$

$$
\begin{aligned}
\theta_1 &\longrightarrow (\theta_1, \theta_2) \in [M(\theta_1)]^2 \longrightarrow x_1 \\
\theta_2 &\longrightarrow (\theta_2, \theta_3) \in [M(\theta_2)]^2 \longrightarrow x_2 \\
\theta_3 &\longrightarrow (\theta_3, \theta_3) \in [M(\theta_3)]^2 \longrightarrow x_3
\end{aligned}
$$

$$\begin{aligned} M(\theta_1) &= \{\theta_1, \theta_2\} \\ M(\theta_2) &= \{\theta_2, \theta_3\} \\ M(\theta_3) &= \{\theta_3\} \end{aligned} \qquad u = \begin{array}{cccc} & x_1 & x_2 & x_3 \\ \theta_1 & \boxed{0} & 1 & 2 \\ \theta_2 & 1 & \boxed{2} & 0 \\ \theta_3 & 0 & 1 & \boxed{2} \end{array}$$

$$\theta_1 \longrightarrow (\theta_1, \theta_2) \in [M(\theta_1)]^2 \longrightarrow x_1$$

$$\theta_2 \longrightarrow (\theta_2, \theta_3) \in [M(\theta_2)]^2 \longrightarrow x_2$$

$$\theta_3 \longrightarrow (\theta_3, \theta_3) \in [M(\theta_3)]^2 \longrightarrow x_3$$

Only θ_3 can be imitated by θ_2, but θ_2 has no incentive to do so

How to construct a more general and appropriate correspondence of messages $\boldsymbol{R(\theta)} \subseteq \boldsymbol{\mathcal{M}}$ associated with $\boldsymbol{M}$ such that a revelation principle applies, and how to define truthful reporting strategies $\boldsymbol{r^* : \Theta \longrightarrow \mathcal{M}}$, with $\boldsymbol{r^*(\theta) \in R(\theta)}$ for all $\boldsymbol{\theta}$?

How to construct a more general and appropriate correspondence of messages $\boldsymbol{R(\theta) \subseteq \mathcal{M}}$ associated with $\boldsymbol{M}$ such that a revelation principle applies, and how to define truthful reporting strategies $\boldsymbol{r^* : \Theta \longrightarrow \mathcal{M}}$, with $\boldsymbol{r^*(\theta) \in R(\theta)}$ for all $\boldsymbol{\theta}$?

From any message correspondence $\boldsymbol{M(\theta)}$ (taking values in any arbitrary set), we construct a certifiability/verifiability configuration

$$\boldsymbol{Y(\theta) \equiv \{M^{-1}(m) : m \in M(\theta)\}}$$

This set is the set of "certificates" or "proofs" available to type $\boldsymbol{\theta}$. Let $\boldsymbol{\mathcal{Y} = \bigcup_\theta Y(\theta)}$ be the set of all certificates

How to construct a more general and appropriate correspondence of messages $R(\theta) \subseteq \mathcal{M}$ associated with M such that a revelation principle applies, and how to define truthful reporting strategies $r^* : \Theta \longrightarrow \mathcal{M}$, with $r^*(\theta) \in R(\theta)$ for all θ?

From any message correspondence $M(\theta)$ (taking values in any arbitrary set), we construct a certifiability/verifiability configuration

$$Y(\theta) \equiv \{M^{-1}(m) : m \in M(\theta)\}$$

This set is the set of "certificates" or "proofs" available to type θ. Let $\mathcal{Y} = \bigcup_\theta Y(\theta)$ be the set of all certificates

The agent can combine certificates (e.g., sending two messages): Let $\mathcal{C}$ be the closure of $\mathcal{Y}$, i.e., the smallest set containing $\mathcal{Y}$ which is closed under intersection, and let

$$C(\theta) = \{c \in \mathcal{C} : \theta \in c\}$$

Example.

$$\begin{aligned} M(\theta_1) &= \{\theta_1, \theta_2\} \\ M(\theta_2) &= \{\theta_2, \theta_3\} \\ M(\theta_3) &= \{\theta_3\} \end{aligned} \quad \Rightarrow \quad \begin{aligned} M^{-1}(\theta_1) &= \{\theta_1\} \\ M^{-1}(\theta_2) &= \{\theta_1, \theta_2\} \\ M^{-1}(\theta_3) &= \{\theta_2, \theta_3\} \end{aligned}$$

so

$$\mathcal{Y} = \{\{\theta_1\}, \{\theta_1, \theta_2\}, \{\theta_2, \theta_3\}\}$$

$$\mathcal{C} = \{\{\theta_1\}, \{\theta_2\}, \{\theta_1, \theta_2\}, \{\theta_2, \theta_3\}\}$$

Complete certification:

$$c^*(\theta) = \bigcap_{c \in C(\theta)} c \;=\; \text{smallest element of } C(\theta)$$

Complete certification:

$$\boldsymbol{c}^*(\boldsymbol{\theta}) = \bigcap_{\boldsymbol{c} \in \boldsymbol{C}(\boldsymbol{\theta})} \boldsymbol{c} \ = \ \text{smallest element of } \boldsymbol{C}(\boldsymbol{\theta})$$

Truthful strategy:

$$\boldsymbol{r}^*(\boldsymbol{\theta}) = (\boldsymbol{\theta}, \boldsymbol{c}^*(\boldsymbol{\theta})) \in \boldsymbol{\Theta} \times \boldsymbol{C}(\boldsymbol{\theta}) \equiv \boldsymbol{R}(\boldsymbol{\theta})$$

Complete certification:

$$c^*(\theta) = \bigcap_{c \in C(\theta)} c \;=\; \text{smallest element of } C(\theta)$$

Truthful strategy:

$$r^*(\theta) = (\theta, c^*(\theta)) \in \Theta \times C(\theta) \equiv R(\theta)$$

Proposition 2 *(Forges and Koessler, 2005) Whatever the message correspondence $M(\theta)$, $\theta \in \Theta$, the decision set X and the utility function $u : X \times \Theta \longrightarrow \mathbb{R}$, the set of allocations that are M-implementable in a general communication system (allowing multiple communication stages, random mechanisms,...) coincides with the set of truthful R-implementable allocations*

In examples 1 and 2

$$\boldsymbol{r^*(\theta_1) = (\theta_1, \{\theta_1\})}$$
$$\boldsymbol{r^*(\theta_2) = (\theta_2, \{\theta_2\})}$$
$$\boldsymbol{r^*(\theta_3) = (\theta_3, \{\theta_2, \theta_3\})}$$

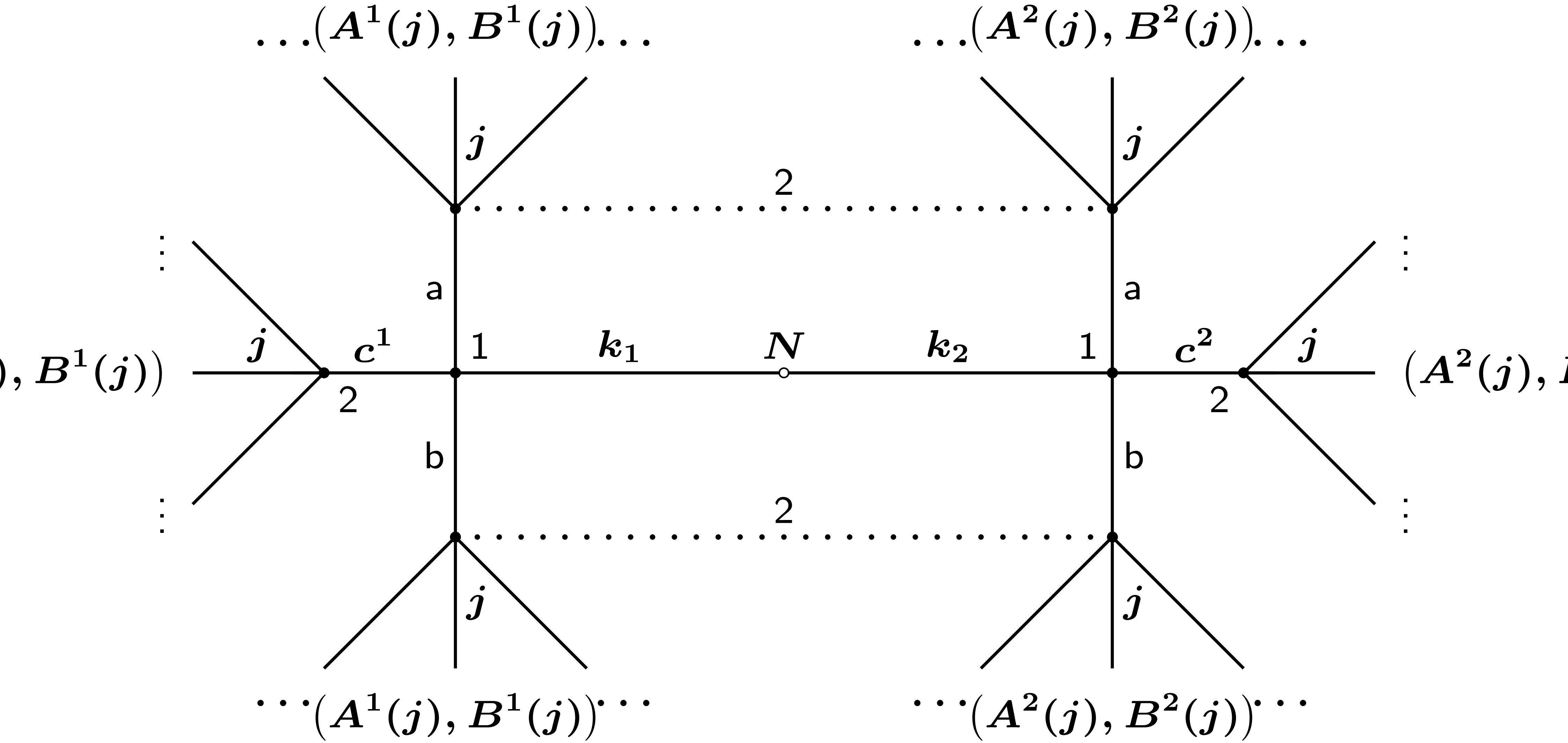
Certifiable Information in Games

Certifiable Information in Games

Unilateral persuasion game $\Gamma_S(p)$: defined as the unilateral cheap talk game $\Gamma_S^0(p)$, but the set of messages of the sender, $M(k)$, depends on his type k

Certifiable Information in Games

Unilateral persuasion game $\Gamma_S(p)$: defined as the unilateral cheap talk game $\Gamma^0_S(p)$, but the set of messages of the sender, $M(k)$, depends on his type k



Examples

In example 3 recalled below the unique NE of the cheap talk game is NR $(\boldsymbol{j_2 \longrightarrow (a, \beta) = ((1,1),2)})$:

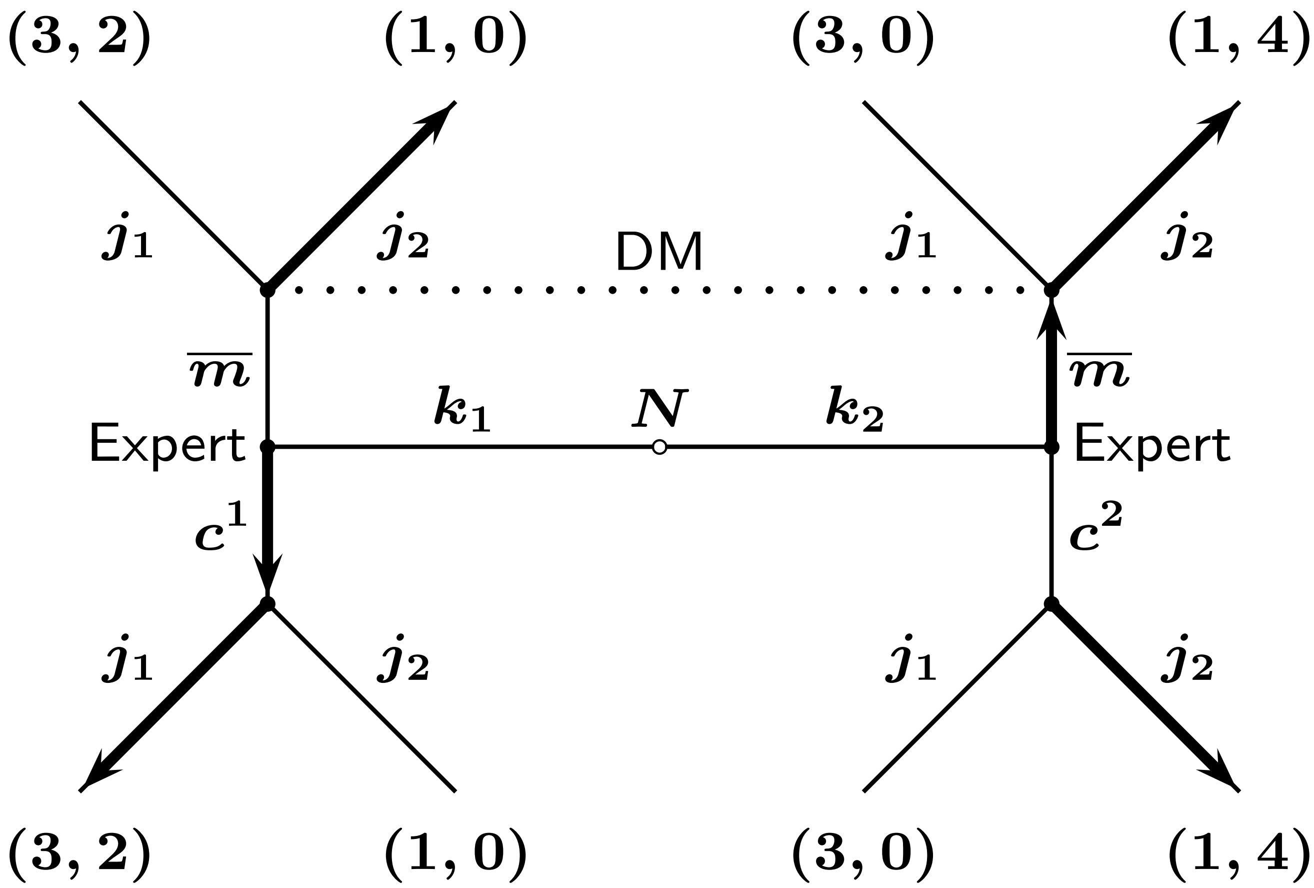
	j_1	j_2	
k_1	$5, 2$	$1, 0$	$p = 1/2$
k_2	$3, 0$	$1, 4$	$(1-p) = 1/2$

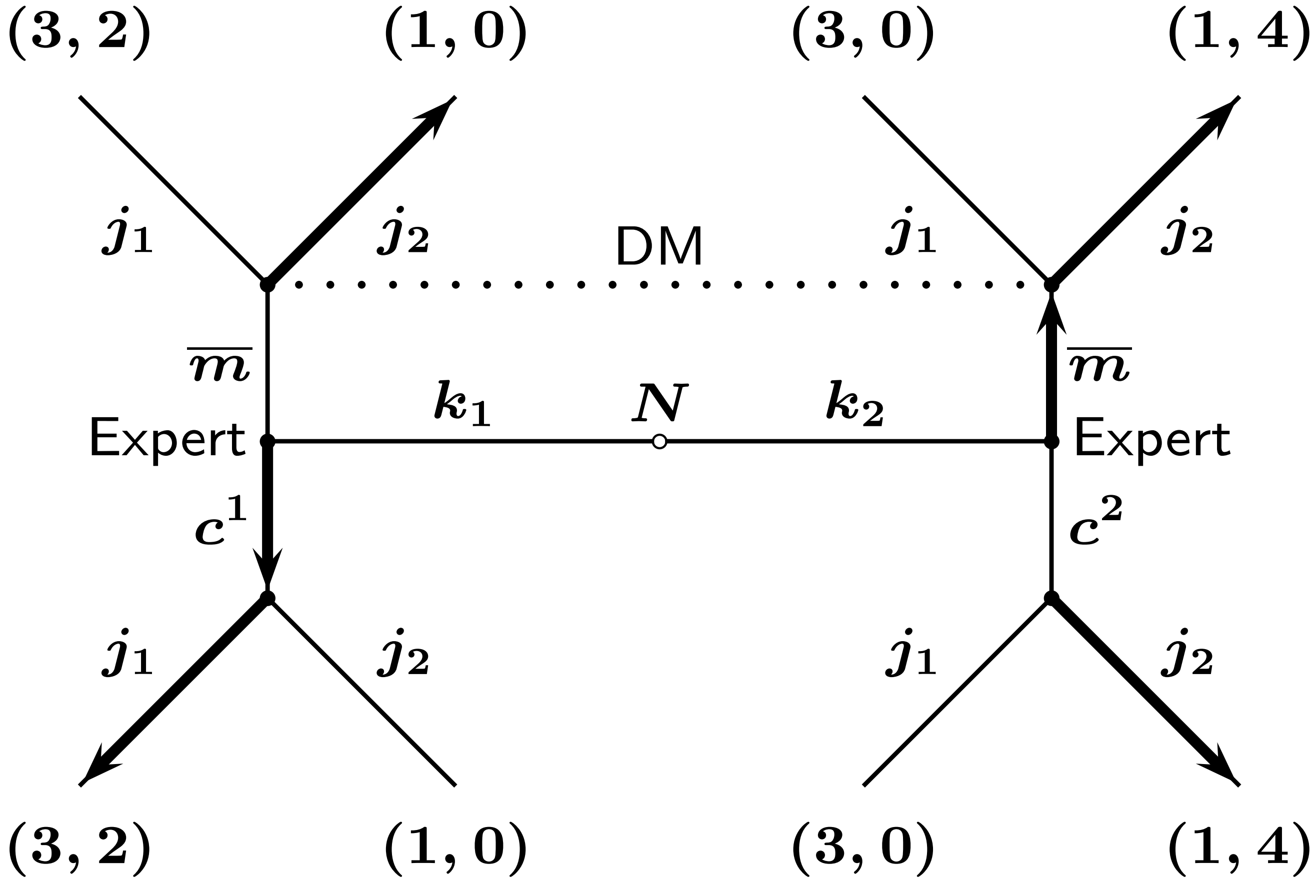
Examples

In example 3 recalled below the unique NE of the cheap talk game is NR $(j_2 \longrightarrow (a, \beta) = ((1,1),2))$:

	j_1	j_2	
k_1	$5, 2$	$1, 0$	$p = 1/2$
k_2	$3, 0$	$1, 4$	$(1-p) = 1/2$

However, if type k_1 is able to prove his type, by sending a message (certificate) $m = c^1$ which is not available to type k_2, then there is a FRE


(3, 2)
(1, 0)
(3, 0)
(1, 4)
j_1
j_2
DM
j_1
j_2
$\overline{m}$
$\overline{m}$
k_1
N
k_2
Expert
Expert
c^1
c^2
j_1
j_2
j_1
j_2
(3, 2)
(1, 0)
(3, 0)
(1, 4)



With certifiable information, there is also a (pure strategy) FRE in the monotonic games 1, 7 and 8, as well as in examples 2 and 5 where there already exists a FRE under cheap talk

On the contrary, examples 4 and 6 don't admit a FRE

Example 10.

	j_1	j_2	j_3	j_4	j_5	
k_1	$5, 0$	$3, 4$	$0, 7$	$4, 9$	$2, 10$	$\Pr[k_1] = 1/2$
k_2	$1, 10$	$3, 9$	$0, 7$	$5, 4$	$6, 0$	$\Pr[k_2] = 1/2$

Example 10.

	j_1	j_2	j_3	j_4	j_5	
k_1	$5,0$	$3,4$	$0,7$	$4,9$	$2,10$	$\Pr[k_1]=1/2$
k_2	$1,10$	$3,9$	$0,7$	$5,4$	$6,0$	$\Pr[k_2]=1/2$

Unique communication equilibrium: non-revealing $(j_3 \longrightarrow ((0,0),7))$

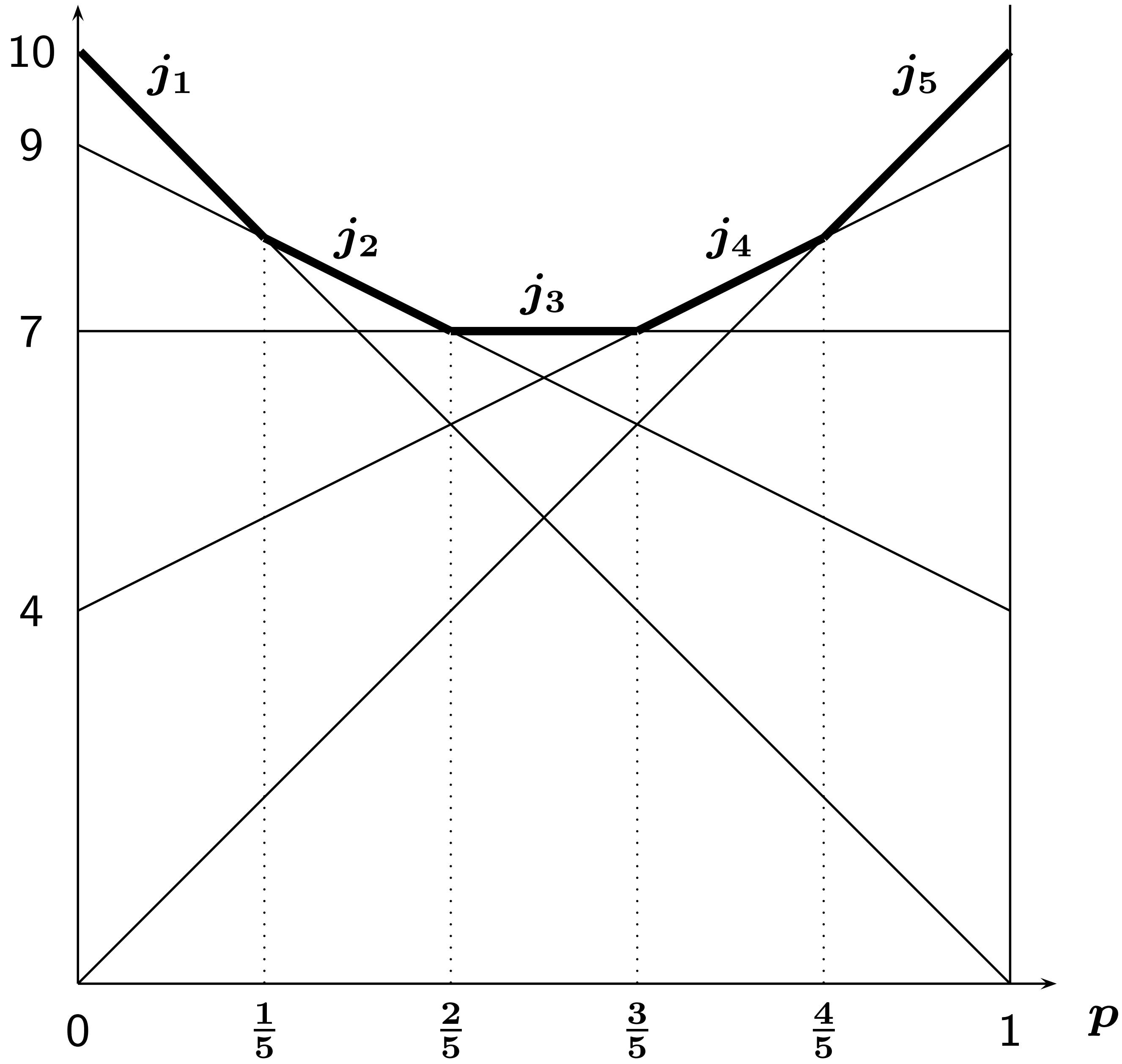
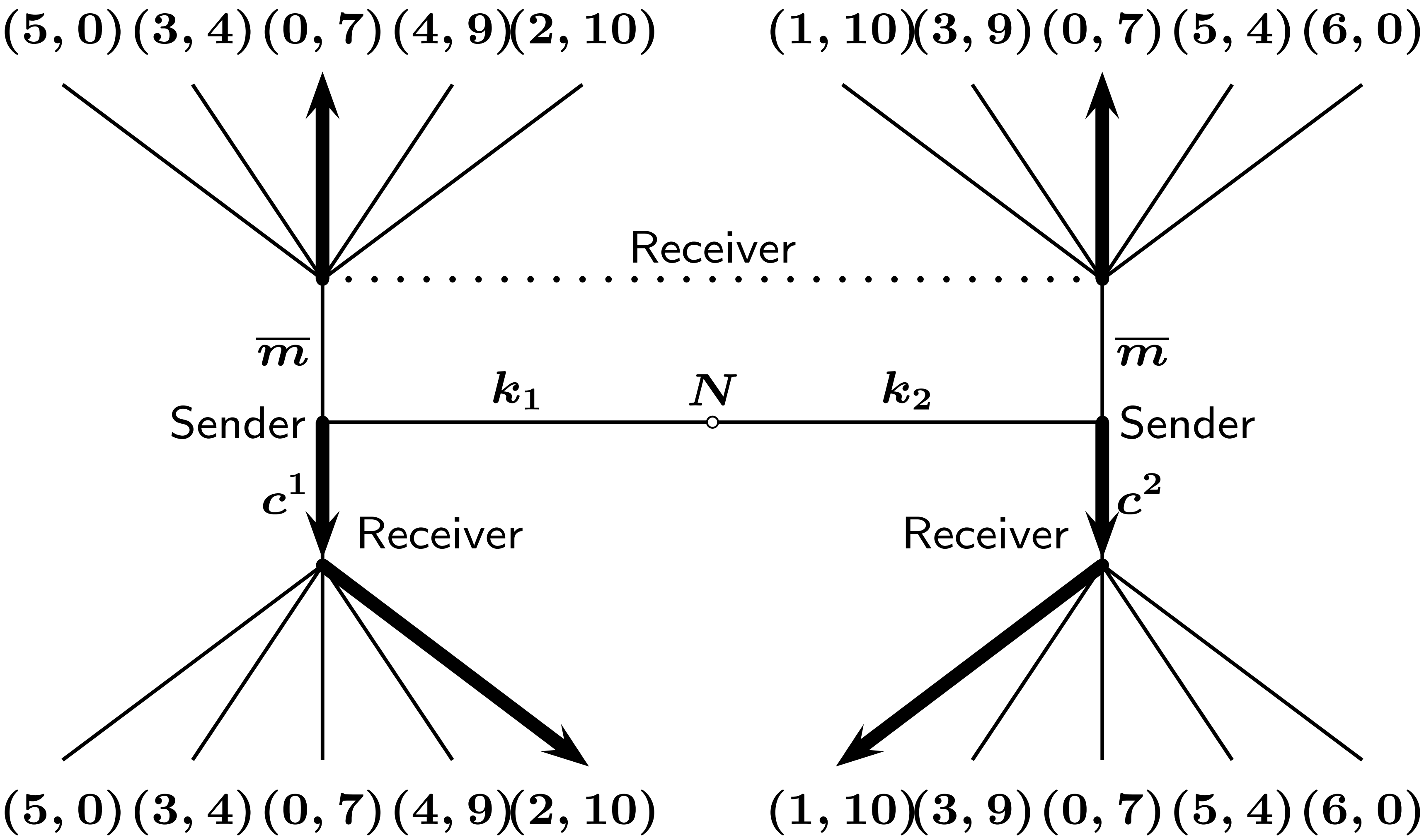


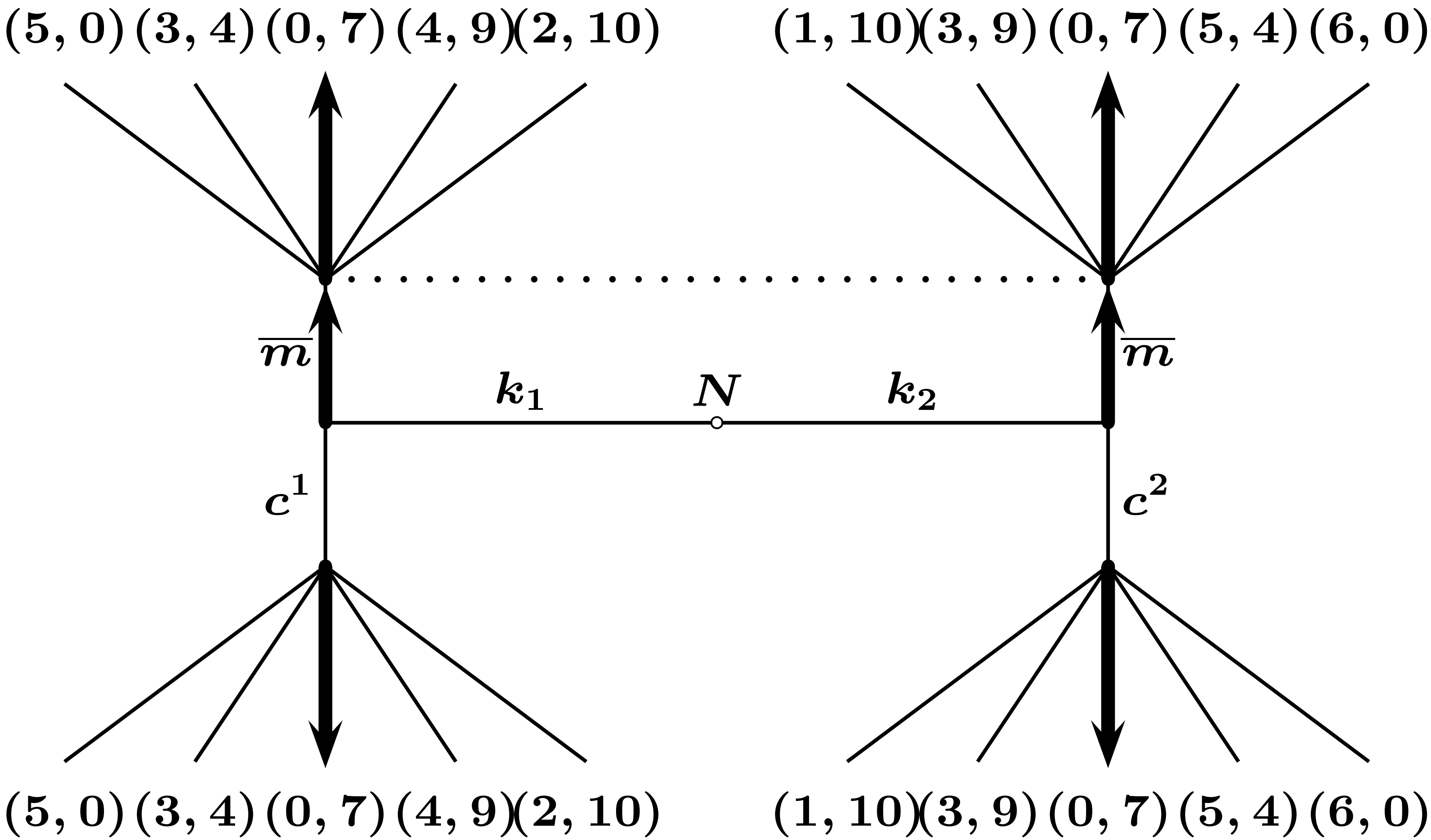
Figure 2: Expected payoffs (fine lines) and best reply expected payoffs (bold lines) for the DM

Fully Revealing Equilibrium



Interim expected payoffs: $(a, \beta) = ((2,1), 10)$

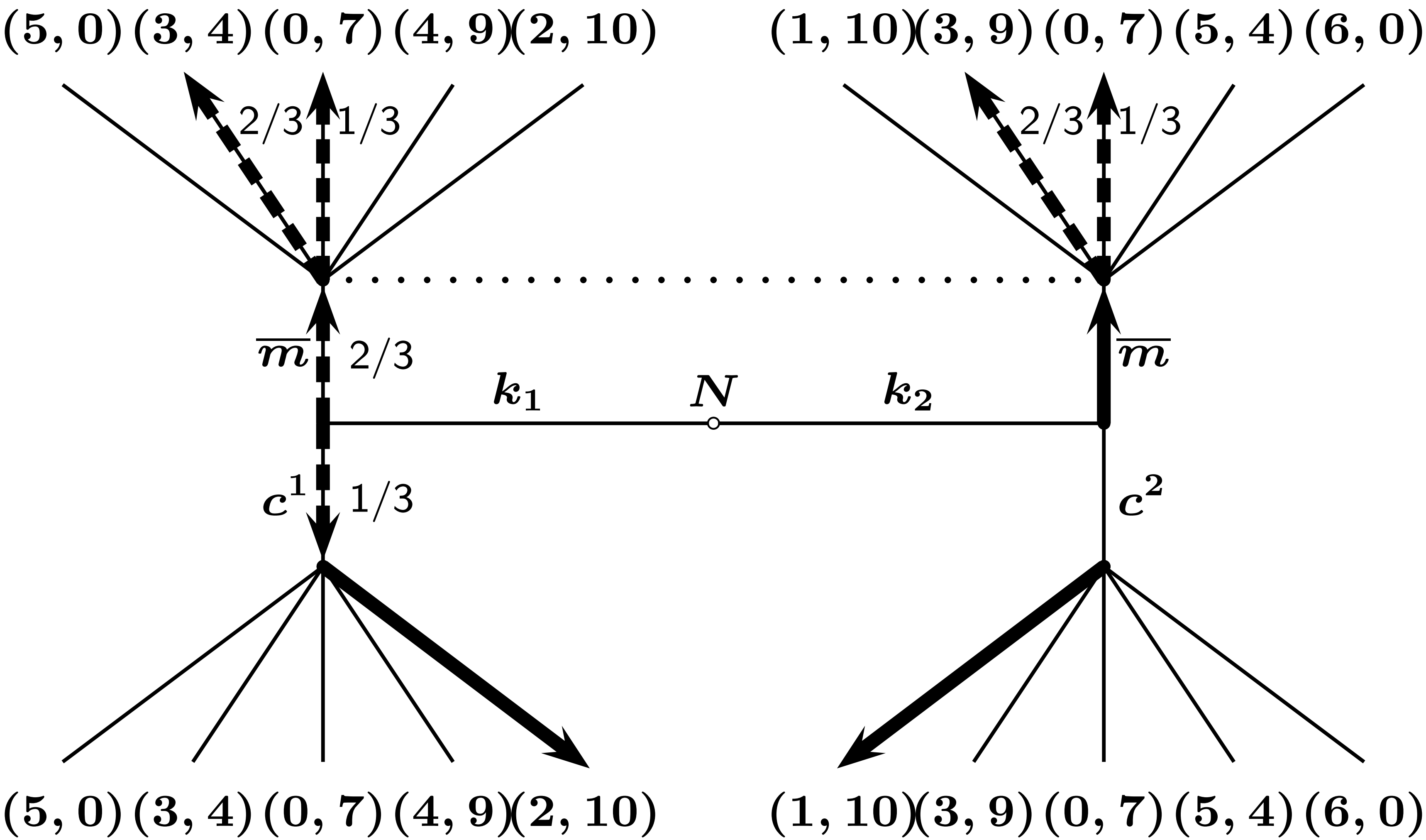
Non-revealing Equilibrium



Interim expected payoffs: $(\boldsymbol{a}, \boldsymbol{\beta}) = ((\mathbf{0}, \mathbf{0}), \mathbf{7})$

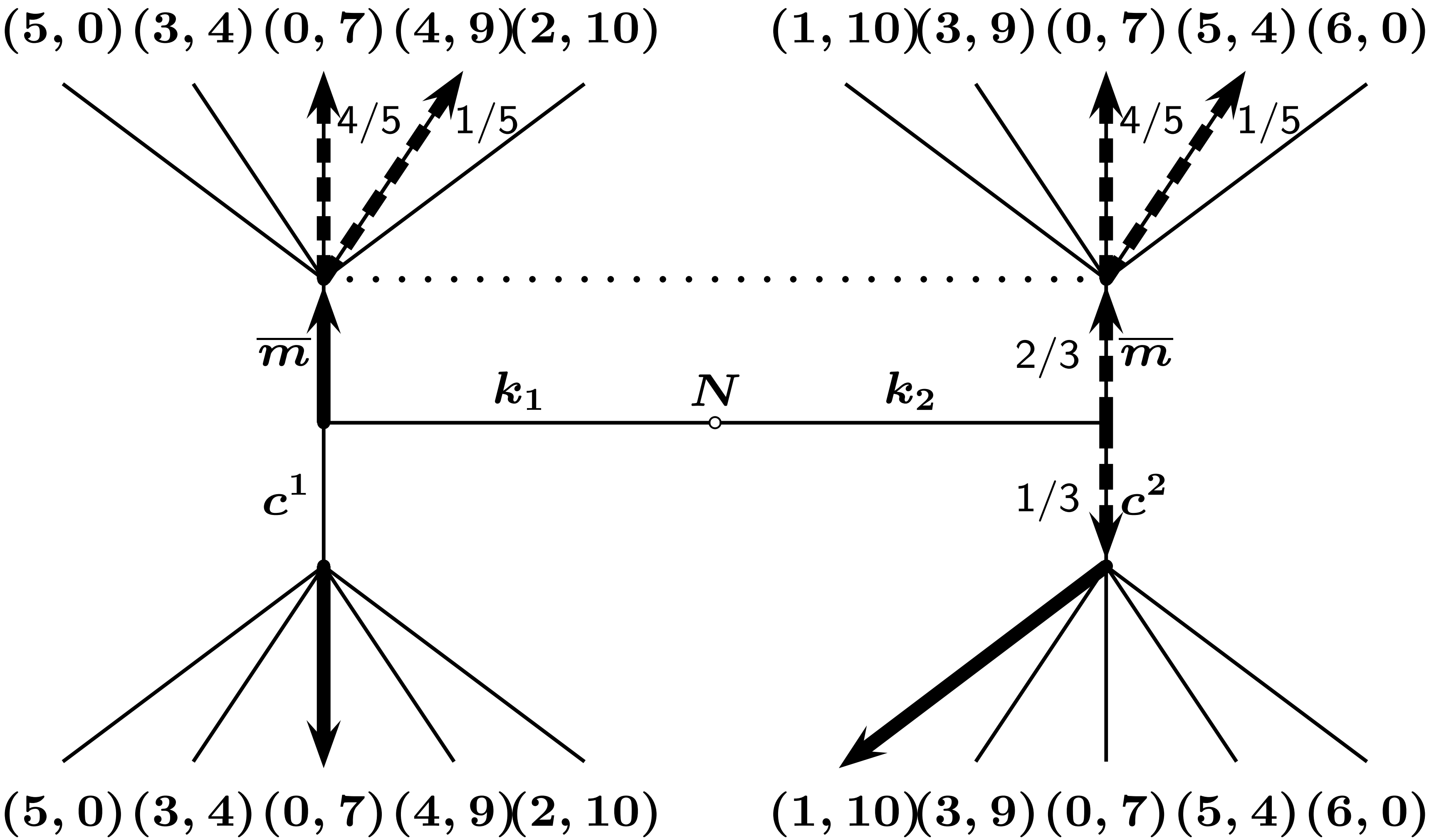
(Note: this NE is not subgame perfect)

Partially Revealing Equilibrium: PRE1



Interim expected payoffs: $(a, \beta) = ((2,2), 7.5)$

Partially Revealing Equilibrium: PRE2



Interim Expected Payoffs: $(a, \beta) = ((4/5, 1), 7.5)$

(Note: This NE is not subgame perfect)

Geometric Characterization of NE payoffs of $\Gamma_S(p)$

Geometric Characterization of NE payoffs of $\Gamma_S(p)$

Recall: **Modified equilibrium payoffs** $\mathcal{E}^+(p)$ of $\Gamma(p)$: the expert can get a payoff higher than his equilibrium when his type has zero probability

Geometric Characterization of NE payoffs of $\Gamma_S(p)$

Recall: **Modified equilibrium payoffs** $\mathcal{E}^+(p)$ of $\Gamma(p)$: the expert can get a payoff higher than his equilibrium when his type has zero probability

➥ $(a, \beta) \in \mathbb{R}^2 \times \mathbb{R}$ such that there exists an optimal mixed action $y \in Y(p)$ of the silent game $\Gamma(p)$ satisfying

Geometric Characterization of NE payoffs of $\Gamma_S(p)$

Recall: **Modified equilibrium payoffs** $\mathcal{E}^+(p)$ of $\Gamma(p)$: the expert can get a payoff higher than his equilibrium when his type has zero probability

➥ $(a, \beta) \in \mathbb{R}^2 \times \mathbb{R}$ such that there exists an optimal mixed action $y \in Y(p)$ of the silent game $\Gamma(p)$ satisfying

(i) $a^k \geq A^k(y)$, for every $k \in K$;

Geometric Characterization of NE payoffs of $\Gamma_S(p)$

Recall: **Modified equilibrium payoffs** $\mathcal{E}^+(p)$ of $\Gamma(p)$: the expert can get a payoff higher than his equilibrium when his type has zero probability

➥ $(a, \beta) \in \mathbb{R}^2 \times \mathbb{R}$ such that there exists an optimal mixed action $y \in Y(p)$ of the silent game $\Gamma(p)$ satisfying

(i) $a^k \geq A^k(y)$, for every $k \in K$;

(ii) $a^1 = A^1(y)$ if $p \neq 0$ and $a^2 = A^2(y)$ if $p \neq 1$;

Geometric Characterization of NE payoffs of $\Gamma_S(p)$

Recall: **Modified equilibrium payoffs** $\mathcal{E}^+(p)$ of $\Gamma(p)$: the expert can get a payoff higher than his equilibrium when his type has zero probability

➥ $(a, \beta) \in \mathbb{R}^2 \times \mathbb{R}$ such that there exists an optimal mixed action $y \in Y(p)$ of the silent game $\Gamma(p)$ satisfying

(i) $a^k \geq A^k(y)$, for every $k \in K$;

(ii) $a^1 = A^1(y)$ if $p \neq 0$ and $a^2 = A^2(y)$ if $p \neq 1$;

(iii) $\beta = p\, B^1(y) + (1-p)\, B^2(y)$.

Geometric Characterization of NE payoffs of $\Gamma_S(p)$

Recall: **Modified equilibrium payoffs** $\mathcal{E}^+(p)$ of $\Gamma(p)$: the expert can get a payoff higher than his equilibrium when his type has zero probability

➥ $(a, \beta) \in \mathbb{R}^2 \times \mathbb{R}$ such that there exists an optimal mixed action $y \in Y(p)$ of the silent game $\Gamma(p)$ satisfying

(i) $a^k \geq A^k(y)$, for every $k \in K$;

(ii) $a^1 = A^1(y)$ if $p \neq 0$ and $a^2 = A^2(y)$ if $p \neq 1$;

(iii) $\beta = p\, B^1(y) + (1 - p)\, B^2(y)$.

Extended equilibrium payoffs $\mathcal{E}^{++}(p)$ of $\Gamma(p)$: the expert can have **any** payoff when his type has zero probability

Geometric Characterization of NE payoffs of $\Gamma_S(p)$

Recall: **Modified equilibrium payoffs** $\mathcal{E}^+(p)$ of $\Gamma(p)$: the expert can get a payoff higher than his equilibrium when his type has zero probability

➥ $(a, \beta) \in \mathbb{R}^2 \times \mathbb{R}$ such that there exists an optimal mixed action $y \in Y(p)$ of the silent game $\Gamma(p)$ satisfying

(i) $a^k \geq A^k(y)$, for every $k \in K$;

(ii) $a^1 = A^1(y)$ if $p \neq 0$ and $a^2 = A^2(y)$ if $p \neq 1$;

(iii) $\beta = p\, B^1(y) + (1-p)\, B^2(y)$.

Extended equilibrium payoffs $\mathcal{E}^{++}(p)$ of $\Gamma(p)$: the expert can have **any** payoff when his type has zero probability

➥ $(a, \beta) \in \mathbb{R}^2 \times \mathbb{R}$ such that there exists $y \in Y(p)$ satisfying (ii) and (iii)

Graph of the extended equilibrium payoff correspondence:

$$\mathbf{gr}\,\boldsymbol{\mathcal{E}}^{++} \equiv \{(\boldsymbol{a}, \boldsymbol{\beta}, \boldsymbol{p}) \in \mathbb{R}^{\mathbf{2}} \times \mathbb{R} \times [\mathbf{0}, \mathbf{1}] : (\boldsymbol{a}, \boldsymbol{\beta}) \in \boldsymbol{\mathcal{E}}^{++}(\boldsymbol{p})\}$$

Graph of the extended equilibrium payoff correspondence:

$$\mathbf{gr}\,\boldsymbol{\mathcal{E}}^{++} \equiv \{(\boldsymbol{a}, \boldsymbol{\beta}, \boldsymbol{p}) \in \mathbb{R}^{\mathbf{2}} \times \mathbb{R} \times [\mathbf{0}, \mathbf{1}] : (\boldsymbol{a}, \boldsymbol{\beta}) \in \boldsymbol{\mathcal{E}}^{++}(\boldsymbol{p})\}$$

Graph of interim individually rational payoffs:

$$\text{INTIR} \equiv \{(\boldsymbol{a}, \boldsymbol{\beta}, \boldsymbol{p}) \in \mathbb{R}^{\mathbf{2}} \times \mathbb{R} \times [\mathbf{0}, \mathbf{1}] : \exists\ \overline{\boldsymbol{y}} \in \boldsymbol{\Delta}(\boldsymbol{J}),\ \boldsymbol{a}^{\boldsymbol{k}} \geq \boldsymbol{A}^{\boldsymbol{k}}(\overline{\boldsymbol{y}})\ \forall\ \boldsymbol{k} \in \boldsymbol{K}\}$$

Graph of the extended equilibrium payoff correspondence:

$$\mathbf{gr}\,\boldsymbol{\mathcal{E}}^{++} \equiv \{(\boldsymbol{a}, \boldsymbol{\beta}, \boldsymbol{p}) \in \mathbb{R}^{\mathbf{2}} \times \mathbb{R} \times [\mathbf{0}, \mathbf{1}] : (\boldsymbol{a}, \boldsymbol{\beta}) \in \boldsymbol{\mathcal{E}}^{++}(\boldsymbol{p})\}$$

Graph of interim individually rational payoffs:

$$\text{INTIR} \equiv \{(\boldsymbol{a}, \boldsymbol{\beta}, \boldsymbol{p}) \in \mathbb{R}^{\mathbf{2}} \times \mathbb{R} \times [\mathbf{0}, \mathbf{1}] : \exists\ \overline{\boldsymbol{y}} \in \boldsymbol{\Delta}(\boldsymbol{J}),\ \boldsymbol{a}^{\boldsymbol{k}} \geq \boldsymbol{A}^{\boldsymbol{k}}(\overline{\boldsymbol{y}})\ \forall\ \boldsymbol{k} \in \boldsymbol{K}\}$$

Forges and Koessler (2007, JET): If every event is certifiable, all Nash equilibrium payoffs of the unilateral persuasion game $\boldsymbol{\Gamma}_{\boldsymbol{S}}(\boldsymbol{p})$ can be geometrically characterized from the graph of the equilibrium payoff correspondence of the silent game

Assumptions:

Assumptions:

- For every $\boldsymbol{k}$ there exists $\boldsymbol{c^k} \in \boldsymbol{M^1}$ such that $\boldsymbol{M^{-1}(c^k) = \{k\}}$

Assumptions:

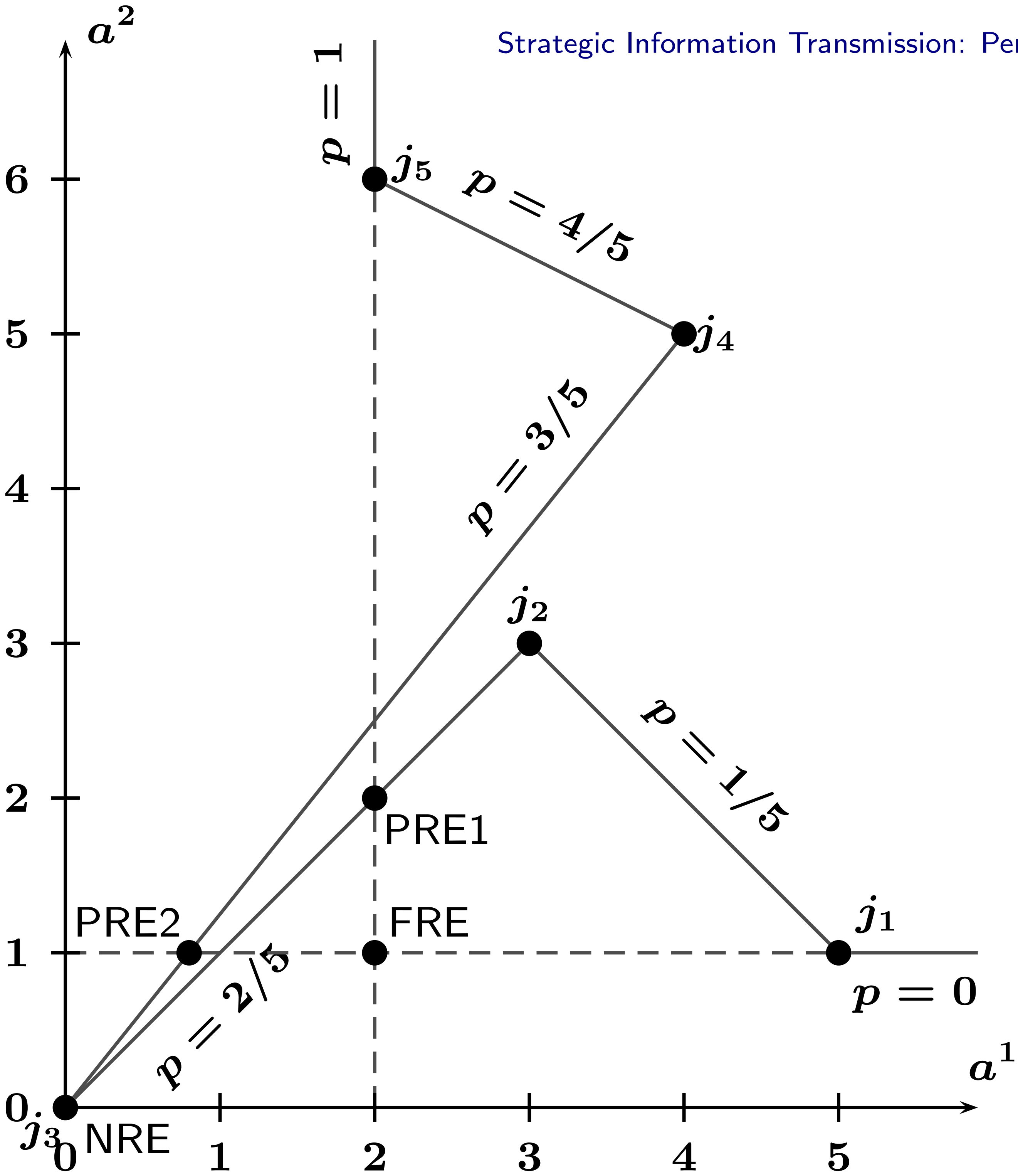
- For every k there exists $c^k \in M^1$ such that $M^{-1}(c^k) = \{k\}$
- $|M(k_1) \cap M(k_2)| \geq 3$

Assumptions:

- For every k there exists $c^k \in M^1$ such that $M^{-1}(c^k) = \{k\}$
- $|M(k_1) \cap M(k_2)| \geq 3$

Theorem (Characterization of $\mathcal{E}_S(p)$) *Let $p \in (0,1)$. A payoff (a, β) is an equilibrium payoff of the unilateral persuasion game $\Gamma_S(p)$ if and only if (a, β, p) belongs to* conv$_a(\mathrm{gr}\, \mathcal{E}^{++}) \cap$ INTIR*, the set of all points obtained by convexifying the set $\mathrm{gr}\, \mathcal{E}^{++}$ in (β, p) while keeping constant and individually rational the expert's payoff, a:*

$$\mathcal{E}_S(p) = \{(a, \beta) \in \mathbb{R}^2 \times \mathbb{R} : (a, \beta, p) \in \text{conv}_a(\mathrm{gr}\, \mathcal{E}^{++}) \cap \text{INTIR}\}.$$


a^2
a^1
6
5
4
3
2
1
0
0
1
2
3
4
5
$p = 1$
j_5
$p = 4/5$
j_4
$p = 3/5$
j_2
$p = 1/5$
PRE1
PRE2
FRE
j_1
$p = 0$
$p = 2/5$
j_3
NRE

Equilibrium Refinement in Persuasion Games

Equilibrium Refinement in Persuasion Games

Contrary to the cheap talk case, a Nash equilibrium in a persuasion game may rely on irrational choices off the equilibrium path

Equilibrium Refinement in Persuasion Games

Contrary to the cheap talk case, a Nash equilibrium in a persuasion game may rely on irrational choices off the equilibrium path

For instance, in example 10, the NRE and the PRE2 are not subgame perfect

Equilibrium Refinement in Persuasion Games

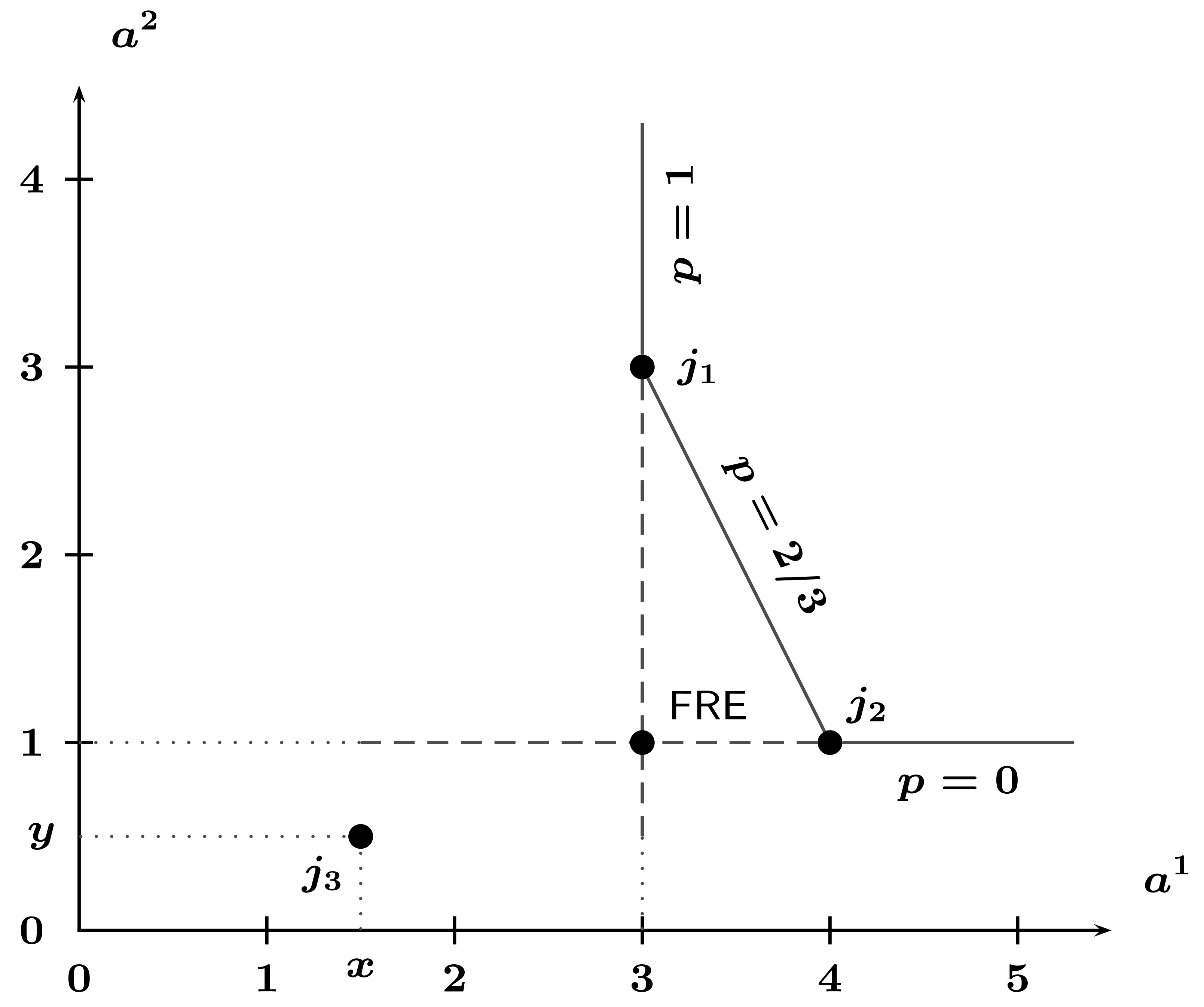
Contrary to the cheap talk case, a Nash equilibrium in a persuasion game may rely on irrational choices off the equilibrium path

For instance, in example 10, the NRE and the PRE2 are not subgame perfect

Similarly, the NRE is not subgame perfect in the persuasion games associated with example 1 when $\boldsymbol{p > 1/4}$, example 2 for every $\boldsymbol{p}$, example 3 when $\boldsymbol{p < 2/3}$, example 5 when $\boldsymbol{p \in (3/8, 5/8)}$, example 7 when $\boldsymbol{p \in (1/3, 2/3)}$, and example 8 when $\boldsymbol{p > 2/5}$

The example below, which is a modified version of example 4 by adding the strictly dominated action j_3, has a subgame perfect FRE when $x \leq 3$ et $y \leq 1$, but it is not a perfect Bayesian equilibrium

	j_1	j_2	j_3	
k_1	$3, 2$	$4, 0$	$x, -1$	p
k_2	$3, 0$	$1, 4$	$y, -1$	$(1 - p)$


a^2
a^1
4
3
2
1
0
y
0
1
x
2
3
4
5
$p = 1$
j_1
$p = 2/3$
FRE
j_2
$p = 0$
j_3

Formally, in the geometric characterization of the theorem, the payoff $\boldsymbol{a} = (\boldsymbol{a}^1, \boldsymbol{a}^2)$ of the expert should also satisfy

$$\exists\ \overline{\boldsymbol{y}}_1 \in \boldsymbol{Y}(1) \text{ t.q. } \boldsymbol{a}^1 \geq \boldsymbol{A}^1(\overline{\boldsymbol{y}}_1)$$
$$\exists\ \overline{\boldsymbol{y}}_2 \in \boldsymbol{Y}(0) \text{ t.q. } \boldsymbol{a}^2 \geq \boldsymbol{A}^2(\overline{\boldsymbol{y}}_2)$$

for a subgame perfect NE ($\Rightarrow$ north-east of FRE)

Formally, in the geometric characterization of the theorem, the payoff $a = (a^1, a^2)$ of the expert should also satisfy

$$\exists\ \overline{y}_1 \in Y(1) \text{ t.q. } a^1 \geq A^1(\overline{y}_1)$$
$$\exists\ \overline{y}_2 \in Y(0) \text{ t.q. } a^2 \geq A^2(\overline{y}_2)$$

for a subgame perfect NE ($\Rightarrow$ north-east of FRE)

and

$$\exists\ p \in \Delta(K),\ \overline{y} \in Y(p) \text{ t.q. } a^k \geq A^k(\overline{y})\ \forall\ k \in K$$

for a perfect Bayesian equilibrium ($\Rightarrow$ north-east of $[j_1, j_2]$)

Formally, in the geometric characterization of the theorem, the payoff $\boldsymbol{a} = (\boldsymbol{a}^1, \boldsymbol{a}^2)$ of the expert should also satisfy

$$\exists\ \overline{\boldsymbol{y}}_1 \in \boldsymbol{Y}(1) \text{ t.q. } \boldsymbol{a}^1 \geq \boldsymbol{A}^1(\overline{\boldsymbol{y}}_1)$$
$$\exists\ \overline{\boldsymbol{y}}_2 \in \boldsymbol{Y}(0) \text{ t.q. } \boldsymbol{a}^2 \geq \boldsymbol{A}^2(\overline{\boldsymbol{y}}_2)$$

for a subgame perfect NE ($\Rightarrow$ north-east of FRE)

and

$$\exists\ \boldsymbol{p} \in \boldsymbol{\Delta}(\boldsymbol{K}),\ \overline{\boldsymbol{y}} \in \boldsymbol{Y}(\boldsymbol{p}) \text{ t.q. } \boldsymbol{a}^k \geq \boldsymbol{A}^k(\overline{\boldsymbol{y}})\ \forall\ \boldsymbol{k} \in \boldsymbol{K}$$

for a perfect Bayesian equilibrium ($\Rightarrow$ north-east of $[\boldsymbol{j}_1, \boldsymbol{j}_2]$)

Now, equilibrium = perfect Bayesian equilibrium

Is Certifiable Information always Better for the DM?

Is Certifiable Information always Better for the DM?

NO. A PBE of a cheap talk game may be better for the DM than **all** PBE of the persuasion game

Is Certifiable Information always Better for the DM?

NO. A PBE of a cheap talk game may be better for the DM than **all** PBE of the persuasion game

Example 11.

	j_1	j_2	j_3	j_4	j_5	
k_1	$2,4$	$1,3$	$0,-5$	$0,-5$	$0,-5$	$\Pr[k_1]=1/3$
k_2	$-1,0$	$3,3$	$1,4$	$4,2$	$2,-5$	$\Pr[k_2]=1/3$
k_3	$-1,0$	$0,-5$	$2,-5$	$2,2$	$1,4$	$\Pr[k_3]=1/3$

Is Certifiable Information always Better for the DM?

NO. A PBE of a cheap talk game may be better for the DM than **all** PBE of the persuasion game

Example 11.

	j_1	j_2	j_3	j_4	j_5	
k_1	$2,4$	$1,3$	$0,-5$	$0,-5$	$0,-5$	$\Pr[k_1]=1/3$
k_2	$-1,0$	$3,3$	$1,4$	$4,2$	$2,-5$	$\Pr[k_2]=1/3$
k_3	$-1,0$	$0,-5$	$2,-5$	$2,2$	$1,4$	$\Pr[k_3]=1/3$

If every type is certifiable, the unique PBE consists for k_2 and k_3 to send the same message, different from k_1's message. The associated payoff for the DM is 8/3

Is Certifiable Information always Better for the DM?

NO. A PBE of a cheap talk game may be better for the DM than **all** PBE of the persuasion game

Example 11.

	j_1	j_2	j_3	j_4	j_5	
k_1	$2, 4$	$1, 3$	$0, -5$	$0, -5$	$0, -5$	$\Pr[k_1] = 1/3$
k_2	$-1, 0$	$3, 3$	$1, 4$	$4, 2$	$2, -5$	$\Pr[k_2] = 1/3$
k_3	$-1, 0$	$0, -5$	$2, -5$	$2, 2$	$1, 4$	$\Pr[k_3] = 1/3$

If every type is certifiable, the unique PBE consists for k_2 and k_3 to send the same message, different from k_1's message. The associated payoff for the DM is 8/3

In the cheap talk game, there is a PBE in which types k_1 and k_2 send the same message, different from k_3's message. The associated payoff for the DM is 10/3

Sceptical strategies in monotonic relationships

Sceptical strategies in monotonic relationships

Monotonic game: For every $\boldsymbol{k}$, $\boldsymbol{A^k(j) > A^k(j') \Leftrightarrow j > j'}$ (or $\boldsymbol{A^k(j) < A^k(j') \Leftrightarrow j > j'}$)

Sceptical strategies in monotonic relationships

Monotonic game: For every $\boldsymbol{k}$, $\boldsymbol{A^k(j) > A^k(j') \Leftrightarrow j > j'}$ (or $\boldsymbol{A^k(j) < A^k(j') \Leftrightarrow j > j'}$)

Assume that every type is certifiable:

$$\forall\, \boldsymbol{k} \in \boldsymbol{K},\ \exists\, \boldsymbol{m} \in \boldsymbol{M(k)},\ \boldsymbol{M^{-1}(m) = \{k\}}$$

Sceptical strategies in monotonic relationships

Monotonic game: For every $\boldsymbol{k}$, $\boldsymbol{A^k(j) > A^k(j') \Leftrightarrow j > j'}$ (or $\boldsymbol{A^k(j) < A^k(j') \Leftrightarrow j > j'}$)

Assume that every type is certifiable:

$$\boldsymbol{\forall\, k \in K,\ \exists\, m \in M(k),\ M^{-1}(m) = \{k\}}$$

Theorem *Every monotonic game in which every type is certifiable has a perfect Bayesian equilibrium which is fully revealing*

Sceptical strategies in monotonic relationships

Monotonic game: For every k, $A^k(j) > A^k(j') \Leftrightarrow j > j'$ (or $A^k(j) < A^k(j') \Leftrightarrow j > j'$)

Assume that every type is certifiable:

$$\forall\, k \in K,\ \exists\, m \in M(k),\ M^{-1}(m) = \{k\}$$

Theorem *Every monotonic game in which every type is certifiable has a perfect Bayesian equilibrium which is fully revealing*

Proof. It suffices to consider the following sceptical strategy for the DM, consisting in choosing the minimal action among the set of actions that a best response for the types compatible with the message sent:

$$\tau(m) = \min\{j \in J : \exists\, k \in M^{-1}(m), j \in \arg\max_{j'} B^k(j')\}$$

With no additional assumption, other equilibrium outcomes may exist

With no additional assumption, other equilibrium outcomes may exist

For instance, in the monotonic example 3, if $p \geq 2/3$, there is a PBE in which the expert always send the same message and the DM chooses action j_1

With no additional assumption, other equilibrium outcomes may exist

For instance, in the monotonic example 3, if $p \geq 2/3$, there is a PBE in which the expert always send the same message and the DM chooses action j_1

The FRE is unique if we assume that $J \subseteq \mathbb{R}$ and $B^k(j)$ is strictly concave in j for every k (Milgrom, 1981; Grossman, 1981; Milgrom and Roberts, 1986)

Persuasion with Type-Dependent Biases (Seidmann and Winter, 1997)

Generalization of the model of Crawford and Sobel (1982):

Persuasion with Type-Dependent Biases (Seidmann and Winter, 1997)

Generalization of the model of Crawford and Sobel (1982):

- Types of the expert: $T = [0, 1]$, with prior $p(t)$

Persuasion with Type-Dependent Biases (Seidmann and Winter, 1997)

Generalization of the model of Crawford and Sobel (1982):

- Types of the expert: $\boldsymbol{T = [0, 1]}$, with prior $\boldsymbol{p(t)}$
- Actions of the DM: $\boldsymbol{A} \subseteq \mathbb{R}$

Persuasion with Type-Dependent Biases (Seidmann and Winter, 1997)

Generalization of the model of Crawford and Sobel (1982):

- Types of the expert: $\boldsymbol{T = [0, 1]}$, with prior $\boldsymbol{p(t)}$
- Actions of the DM: $\boldsymbol{A} \subseteq \mathbb{R}$
- Utility of the expert: $\boldsymbol{u_1(a; t)}$

Persuasion with Type-Dependent Biases (Seidmann and Winter, 1997)

Generalization of the model of Crawford and Sobel (1982):

- Types of the expert: $\boldsymbol{T = [0, 1]}$, with prior $\boldsymbol{p(t)}$
- Actions of the DM: $\boldsymbol{A} \subseteq \mathbb{R}$
- Utility of the expert: $\boldsymbol{u_1(a; t)}$
- Utility of the DM: $\boldsymbol{u_2(a; t)}$

Persuasion with Type-Dependent Biases (Seidmann and Winter, 1997)

Generalization of the model of Crawford and Sobel (1982):

- Types of the expert: $\boldsymbol{T = [0, 1]}$, with prior $\boldsymbol{p(t)}$
- Actions of the DM: $\boldsymbol{A} \subseteq \mathbb{R}$
- Utility of the expert: $\boldsymbol{u_1(a; t)}$
- Utility of the DM: $\boldsymbol{u_2(a; t)}$
- Messages of the expert of type $\boldsymbol{t} \in \boldsymbol{T}$: $\boldsymbol{M(t)}$

Persuasion with Type-Dependent Biases (Seidmann and Winter, 1997)

Generalization of the model of Crawford and Sobel (1982):

- Types of the expert: $T = [0, 1]$, with prior $p(t)$
- Actions of the DM: $A \subseteq \mathbb{R}$
- Utility of the expert: $u_1(a; t)$
- Utility of the DM: $u_2(a; t)$
- Messages of the expert of type $t \in T$: $M(t)$

A set of type $L \subseteq T$ is said certifiable if there is a message m, denoted by "L", which certifies L: $\exists\ m \in M$ s.t. $M^{-1}(m) = L$

Persuasion with Type-Dependent Biases (Seidmann and Winter, 1997)

Generalization of the model of Crawford and Sobel (1982):

- Types of the expert: $T = [0, 1]$, with prior $p(t)$
- Actions of the DM: $A \subseteq \mathbb{R}$
- Utility of the expert: $u_1(a; t)$
- Utility of the DM: $u_2(a; t)$
- Messages of the expert of type $t \in T$: $M(t)$

A set of type $L \subseteq T$ is said certifiable if there is a message m, denoted by "L", which certifies L: $\exists\ m \in M$ s.t. $M^{-1}(m) = L$

Assumption: $M^{-1}(m)$ is closed, and every singleton $\{t\}$ is certifiable

Assumption A1. (Preference of the DM) For every $t \in T$, $u_2(\cdot; t)$ is concave in a, and

$$a_2^*(t) = \arg\max_{a \in A} u_2(a; t)$$

is unique for every t, continuous and strictly concave in t

Assumption A1. (Preference of the DM) For every $t \in T$, $u_2(\cdot; t)$ is concave in a, and

$$a_2^*(t) = \arg\max_{a \in A} u_2(a; t)$$

is unique for every t, continuous and strictly concave in t

Assumption A2. (Preference of the expert) For every $t \in T$, $u_1(\cdot; t)$ is strictly concave in a, and

$$a_1^*(t) = \arg\max_{a \in A} u_1(a; t)$$

is unique for every t, C^1 and strictly increasing in t

Assumption A1. (Preference of the DM) For every $t \in T$, $u_2(\cdot; t)$ is concave in a, and

$$a_2^*(t) = \arg\max_{a \in A} u_2(a; t)$$

is unique for every t, continuous and strictly concave in t

Assumption A2. (Preference of the expert) For every $t \in T$, $u_1(\cdot; t)$ is strictly concave in a, and

$$a_1^*(t) = \arg\max_{a \in A} u_1(a; t)$$

is unique for every t, C^1 and strictly increasing in t

Remarks.

- The assumptions of the **general** model of Crawford and Sobel (1982) are stronger: here, the bias $D(t) = a_2^*(t) - a_1^*(t)$ is type dependent and may change sign

- All results below apply (and are easy to prove) if we replace A2 by the monotonicity assumption, i.e., $u_1(\cdot; t)$ strictly increasing in a (so that $a_1^*(t)$ does not depend on t). See Milgrom (1981), Milgrom and Roberts (1986)

- All results below apply (and are easy to prove) if we replace A2 by the monotonicity assumption, i.e., $u_1(\cdot; t)$ strictly increasing in a (so that $a_1^*(t)$ does not depend on t). See Milgrom (1981), Milgrom and Roberts (1986)

Simple class of preferences satisfying A1 and A2 :

$$\begin{cases} u_1(a;t) = -[a - a_1^*(t)]^2, & a_1^*(t) = \alpha + \beta\, t \\ u_2(a;t) = -[a - a_2^*(t)]^2, & a_2^*(t) = \gamma + \delta\, t \end{cases}$$

where β, $\delta > 0$

- All results below apply (and are easy to prove) if we replace A2 by the monotonicity assumption, i.e., $u_1(\cdot;t)$ strictly increasing in a (so that $a_1^*(t)$ does not depend on t). See Milgrom (1981), Milgrom and Roberts (1986)

Simple class of preferences satisfying A1 and A2 :

$$\begin{cases} u_1(a;t) = -[a - a_1^*(t)]^2, & a_1^*(t) = \alpha + \beta\, t \\ u_2(a;t) = -[a - a_2^*(t)]^2, & a_2^*(t) = \gamma + \delta\, t \end{cases}$$

where β, $\delta > 0$

Example of Crawford and Sobel (1982): $\alpha = b$, $\beta = \delta = 1$, $\gamma = 0$

A1 + individual rationality $\Rightarrow$ the DM plays $\boldsymbol{a_1^*(l)}$ for some $\boldsymbol{l} \in \mathbf{co}(\boldsymbol{L})$ when he receives message "$\boldsymbol{L}$" (along and off the equilibrium path)

A1 + individual rationality $\Rightarrow$ the DM plays $a_1^*(l)$ for some $l \in \mathbf{co}(L)$ when he receives message "L" (along and off the equilibrium path)

Definition $l \in T$ is a worst case inference for message "L", $l \in \mathbf{wci}(L)$, if $l \in \mathbf{co}(L)$ and

$$u(a_2^*(t); t) \geq u(a_2^*(l); t), \quad \forall\, l \in L$$

A1 + individual rationality $\Rightarrow$ the DM plays $\boldsymbol{a_1^*(l)}$ for some $\boldsymbol{l \in \mathbf{co}(L)}$ when he receives message "$\boldsymbol{L}$" (along and off the equilibrium path)

Definition $\boldsymbol{l \in T}$ is a worst case inference for message "$\boldsymbol{L}$", $\boldsymbol{l \in \mathbf{wci}(L)}$, if $\boldsymbol{l \in \mathbf{co}(L)}$ and

$$\boldsymbol{u(a_2^*(t); t) \geq u(a_2^*(l); t), \quad \forall\, l \in L}$$

Proposition 3 *Under assumption A1 there is a FRE iff every certifiable subset of types has a worst case inference*

Proof. ✍ By definition □

A1 + individual rationality $\Rightarrow$ the DM plays $a_1^*(l)$ for some $l \in \mathbf{co}(L)$ when he receives message "L" (along and off the equilibrium path)

Definition $l \in T$ is a worst case inference for message "L", $l \in \mathbf{wci}(L)$, if $l \in \mathbf{co}(L)$ and

$$u(a_2^*(t);t) \geq u(a_2^*(l);t), \quad \forall\, l \in L$$

Proposition 3 *Under assumption A1 there is a FRE iff every certifiable subset of types has a worst case inference*

Proof. ✍ By definition □

- Let $D(t) = a_2^*(t) - a_1^*(t)$

A1 + individual rationality $\Rightarrow$ the DM plays $a_1^*(l)$ for some $l \in \mathbf{co}(L)$ when he receives message "L" (along and off the equilibrium path)

Definition $l \in T$ is a worst case inference for message "L", $l \in \mathbf{wci}(L)$, if $l \in \mathbf{co}(L)$ and

$$u(a_2^*(t); t) \geq u(a_2^*(l); t), \quad \forall\, l \in L$$

Proposition 3 *Under assumption A1 there is a FRE iff every certifiable subset of types has a worst case inference*

Proof. ✍ By definition □

- Let $D(t) = a_2^*(t) - a_1^*(t)$

A1 + A2 $\Rightarrow$ $D(t)$ is well defined and continuous

A1 + individual rationality $\Rightarrow$ the DM plays $a_1^*(l)$ for some $l \in \mathbf{co}(L)$ when he receives message "L" (along and off the equilibrium path)

Definition $l \in T$ is a worst case inference for message "L", $l \in \mathbf{wci}(L)$, if $l \in \mathbf{co}(L)$ and

$$u(a_2^*(t); t) \geq u(a_2^*(l); t), \quad \forall\, l \in L$$

Proposition 3 *Under assumption A1 there is a FRE iff every certifiable subset of types has a worst case inference*

Proof. ✍ By definition □

- Let $D(t) = a_2^*(t) - a_1^*(t)$

A1 + A2 $\Rightarrow D(t)$ is well defined and continuous

- For every closed $L \subseteq T$, let

$$L_+ = \max\{t \in L\} \qquad L_- = \min\{t \in L\}$$

Theorem *If A1, A2 and either*

(a) $D(t)$ *does not change sign on* T*, or*

(b) $D(t)$ *changes sign only once on* T*, and* $D(0) > 0$

then there is a FRE, and every equilibrium is FR

Theorem *If A1, A2 and either*

(a) $D(t)$ *does not change sign on* T*, or*

(b) $D(t)$ *changes sign only once on* T*, and* $D(0) > 0$

then there is a FRE, and every equilibrium is FR

Proof.

✍ *Existence.* Easy. In case (a) with $D(t) \leq 0$, $L_- \in \mathbf{wci}(L)$; in case (a) with $D(t) \geq 0$, $L_+ \in \mathbf{wci}(L)$; in case (b), $t^* \in \mathbf{wci}(L)$, where $D(t^*) = 0$ □

Theorem *If A1, A2 and either*

(a) $\boldsymbol{D(t)}$ *does not change sign on* $\boldsymbol{T}$*, or*

(b) $\boldsymbol{D(t)}$ *changes sign only once on* $\boldsymbol{T}$*, and* $\boldsymbol{D(0) > 0}$

then there is a FRE, and every equilibrium is FR

Proof.

✍ *Existence.* Easy. In case (a) with $\boldsymbol{D(t) \leq 0}$, $\boldsymbol{L_- \in \mathrm{wci}(L)}$; in case (a) with $\boldsymbol{D(t) \geq 0}$, $\boldsymbol{L_+ \in \mathrm{wci}(L)}$; in case (b), $\boldsymbol{t^* \in \mathrm{wci}(L)}$, where $\boldsymbol{D(t^*) = 0}$ □

Examples.

- General model of Crawford and Sobel (1982), where $\boldsymbol{D(t) > 0}$ or $\boldsymbol{D(t) < 0}$

- Previous parametric class:

$$\boldsymbol{D(t) = a_2^*(t) - a_1^*(t) = (\gamma - \alpha) + (\delta - \beta)\ t}$$

- Previous parametric class:

$$\boldsymbol{D(t) = a_2^*(t) - a_1^*(t) = (\gamma - \alpha) + (\delta - \beta)\, t}$$

If $\boldsymbol{\beta \geq \delta}$ then (a) or (b) so there is a unique, FRE

- Previous parametric class:

$$D(t) = a_2^*(t) - a_1^*(t) = (\gamma - \alpha) + (\delta - \beta)\ t$$

If $\beta \geq \delta$ then (a) or (b) so there is a unique, FRE

If $\beta < \delta$ then (a) is satisfied iff $\alpha - \gamma \notin (0, \delta - \beta)$

- Previous parametric class:

$$D(t) = a_2^*(t) - a_1^*(t) = (\gamma - \alpha) + (\delta - \beta)\, t$$

If $\beta \geq \delta$ then (a) or (b) so there is a unique, FRE

If $\beta < \delta$ then (a) is satisfied iff $\alpha - \gamma \notin (0, \delta - \beta)$

The theorem does not apply when $\alpha - \gamma \in (0, \delta - \beta)$, i.e., when $D(t)$ is increasing and changes sign, for example when $\alpha = \beta = 1$, $\gamma = 0$, $\delta = 5$, $D(t) = -1 + 4\, t$

- Previous parametric class:

$$D(t) = a_2^*(t) - a_1^*(t) = (\gamma - \alpha) + (\delta - \beta)\ t$$

If $\beta \geq \delta$ then (a) or (b) so there is a unique, FRE

If $\beta < \delta$ then (a) is satisfied iff $\alpha - \gamma \notin (0, \delta - \beta)$

The theorem does not apply when $\alpha - \gamma \in (0, \delta - \beta)$, i.e., when $D(t)$ is increasing and changes sign, for example when $\alpha = \beta = 1$, $\gamma = 0$, $\delta = 5$, $D(t) = -1 + 4\ t$

However, there is still a FRE, as shown in the next theorem, but it is not unique and the worst case inference is not obvious

Assumption A3. (Preference of the expert: "Single crossing") If

$$u_1(\overline{a};\underline{t}) \geq u_1(\underline{a};\underline{t}), \quad \text{where } \overline{a} > \underline{a}$$

then, for every $\overline{t} > \underline{t}$ we have

$$u_1(\overline{a};\overline{t}) > u_1(\underline{a};\overline{t})$$

Assumption A3. (Preference of the expert: "Single crossing") If

$$u_1(\overline{a};\underline{t}) \geq u_1(\underline{a};\underline{t}), \quad \text{where } \overline{a} > \underline{a}$$

then, for every $\overline{t} > \underline{t}$ we have

$$u_1(\overline{a};\overline{t}) > u_1(\underline{a};\overline{t})$$

Property. Under A2, if $u_1(\cdot;t)$ is symmetric around $a_1^*(t)$ for every t then A3 is satisfied (particular case: quadratic preferences)

Assumption A3. (Preference of the expert: "Single crossing") If

$$\boldsymbol{u_1(\overline{a};\underline{t}) \geq u_1(\underline{a};\underline{t})}, \quad \text{where } \boldsymbol{\overline{a} > \underline{a}}$$

then, for every $\boldsymbol{\overline{t} > \underline{t}}$ we have

$$\boldsymbol{u_1(\overline{a};\overline{t}) > u_1(\underline{a};\overline{t})}$$

Property. Under A2, if $\boldsymbol{u_1(\cdot;t)}$ is symmetric around $\boldsymbol{a_1^*(t)}$ for every $\boldsymbol{t}$ then A3 is satisfied (particular case: quadratic preferences)

Theorem *Under A1, A2 and A3 there is a FRE, but may not be unique*

The theorem applies with quadratic preferences, in particular in the previous example when $D(t) = -1 + 4\ t$ is increasing:

$$a_1^*(t) = \alpha + \beta\ t = 1 + t$$

$$a_2^*(t) = \gamma + \delta\ t = 5\ t$$

The theorem applies with quadratic preferences, in particular in the previous example when $D(t) = -1 + 4\ t$ is increasing:

$$a_1^*(t) = \alpha + \beta\ t = 1 + t$$
$$a_2^*(t) = \gamma + \delta\ t = 5\ t$$

However, if for instance the prior p is uniform on T, there is also a partially revealing equilibrium (se Seidmann and Winter, 1997)

Long Persuasion Games

Long Persuasion Games

In the unilateral persuasion game associated with Example 10 recalled below

	j_1	j_2	j_3	j_4	j_5	
k_1	$5, 0$	$3, 4$	$0, 7$	$4, 9$	$2, 10$	$\Pr[k_1] = 1/2$
k_2	$1, 10$	$3, 9$	$0, 7$	$5, 4$	$6, 0$	$\Pr[k_2] = 1/2$

the highest payoff for the expert is $(2, 2)$ at the partially revealing equilibrium PRE1

Long Persuasion Games

In the unilateral persuasion game associated with Example 10 recalled below

	j_1	j_2	j_3	j_4	j_5	
k_1	$5, 0$	$3, 4$	$0, 7$	$4, 9$	$2, 10$	$\Pr[k_1] = 1/2$
k_2	$1, 10$	$3, 9$	$0, 7$	$5, 4$	$6, 0$	$\Pr[k_2] = 1/2$

the highest payoff for the expert is $(2, 2)$ at the partially revealing equilibrium PRE1

However, in the 3-stage bilateral persuasion game, there is an equilibrium in which the expert can get $(3, 3)$ by delaying information certification

Stage 1: Signaling

Stage 1: Signaling

The expert sends message a or b with a type dependent positive probability

Stage 1: Signaling

The expert sends message a or b with a type dependent positive probability

Equilibrium condition: he must be indifferent between sending a or b, whatever his type

Stage 1: Signaling

The expert sends message a or b with a type dependent positive probability

Equilibrium condition: he must be indifferent between sending a or b, whatever his type

Stage 2: Jointly controlled lottery (JCL)

Stage 1: Signaling

The expert sends message a or b with a type dependent positive probability

Equilibrium condition: he must be indifferent between sending a or b, whatever his type

Stage 2: Jointly controlled lottery (JCL)

Both players decide jointly on how to continue the game

Stage 1: Signaling

The expert sends message a or b with a type dependent positive probability

Equilibrium condition: he must be indifferent between sending a or b, whatever his type

Stage 2: Jointly controlled lottery (JCL)

Both players decide jointly on how to continue the game

Stage 3: Possible certification

Stage 1: Signaling

The expert sends message a or b with a type dependent positive probability

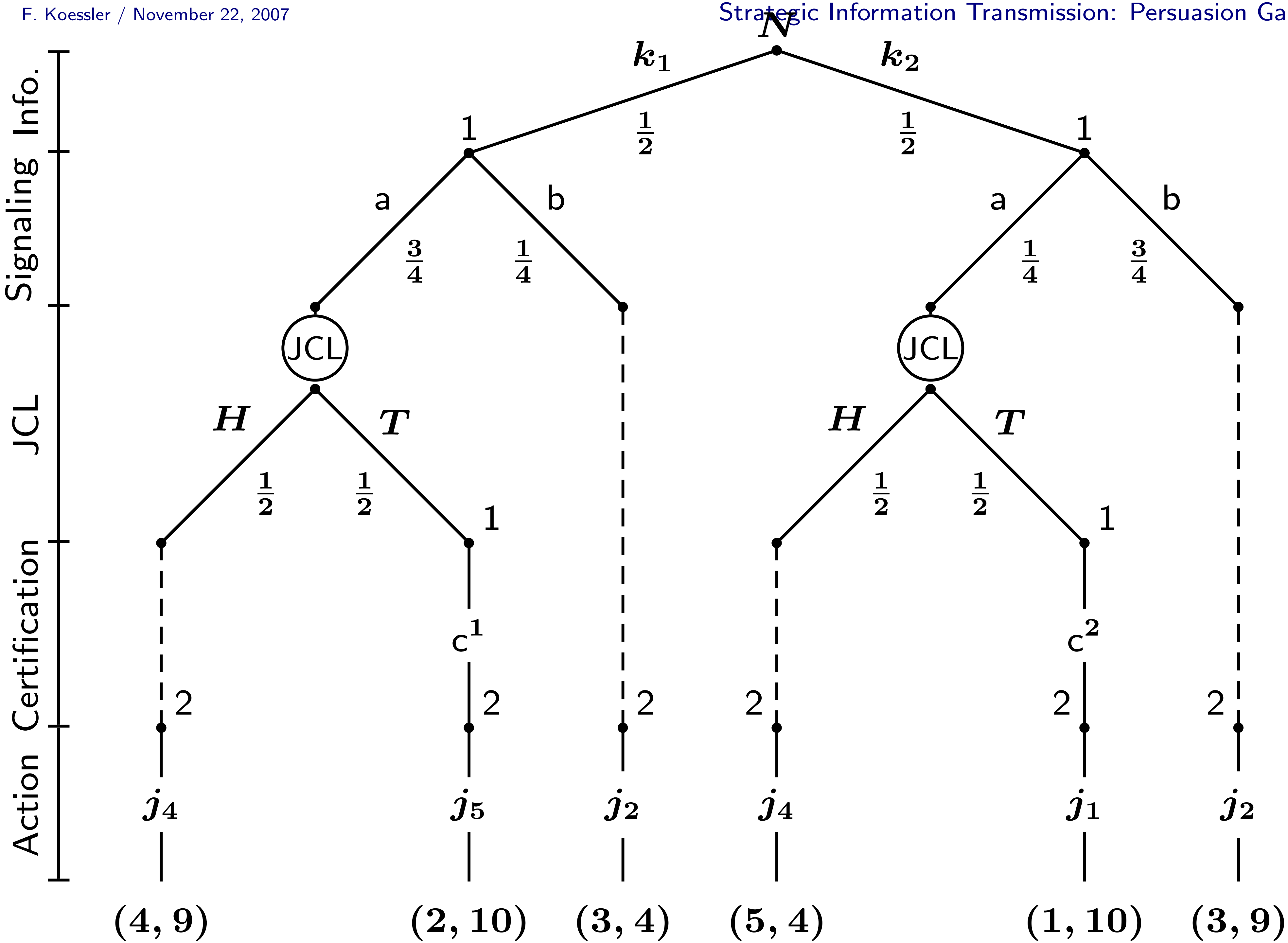
Equilibrium condition: he must be indifferent between sending a or b, whatever his type

Stage 2: Jointly controlled lottery (JCL)

Both players decide jointly on how to continue the game

Stage 3: Possible certification

According to the outcome of the JCL, either P2 makes his decision immediately or P1 first fully certifies his type


N
k1
k2
1/2
1/2
1
1
a
b
a
b
3/4
1/4
1/4
3/4
JCL
JCL
H
T
H
T
1/2
1/2
1/2
1/2
1
1
c1
c2
2
2
2
2
2
2
j4
j5
j2
j4
j1
j2
(4, 9)
(2, 10)
(3, 4)
(5, 4)
(1, 10)
(3, 9)
Signaling Info.
JCL
Certification
Action

$\boldsymbol{\Gamma_n(p)}$: Information and actions phases as in the signalling game $\boldsymbol{\Gamma_S(p)}$ but

- Bilateral communication. Player 2's message set $\boldsymbol{M^2}$, $|\boldsymbol{M^2}| \geq \boldsymbol{2}$
- $\boldsymbol{n \geq 1}$ communication rounds, perfect monitoring

$\boldsymbol{\Gamma_n(p)}$: Information and actions phases as in the signalling game $\boldsymbol{\Gamma_S(p)}$ but

- Bilateral communication. Player 2's message set $\boldsymbol{M^2}$, $|\boldsymbol{M^2}| \geq \boldsymbol{2}$
- $\boldsymbol{n \geq 1}$ communication rounds, perfect monitoring

Information phase — Talking phase ($\boldsymbol{n \geq 1}$ rounds) — Action phase

Expert learns $\boldsymbol{k \in K}$ — Both send $\boldsymbol{(m_t^1, m_t^2) \in M(k) \times M^2}$ $\boldsymbol{(t = 1, \ldots n)}$ — DM chooses $\boldsymbol{j \in J}$

$\boldsymbol{\Gamma_n(p)}$: Information and actions phases as in the signalling game $\boldsymbol{\Gamma_S(p)}$ but

- Bilateral communication. Player 2's message set $\boldsymbol{M^2}$, $|\boldsymbol{M^2}| \geq \boldsymbol{2}$
- $\boldsymbol{n \geq 1}$ communication rounds, perfect monitoring

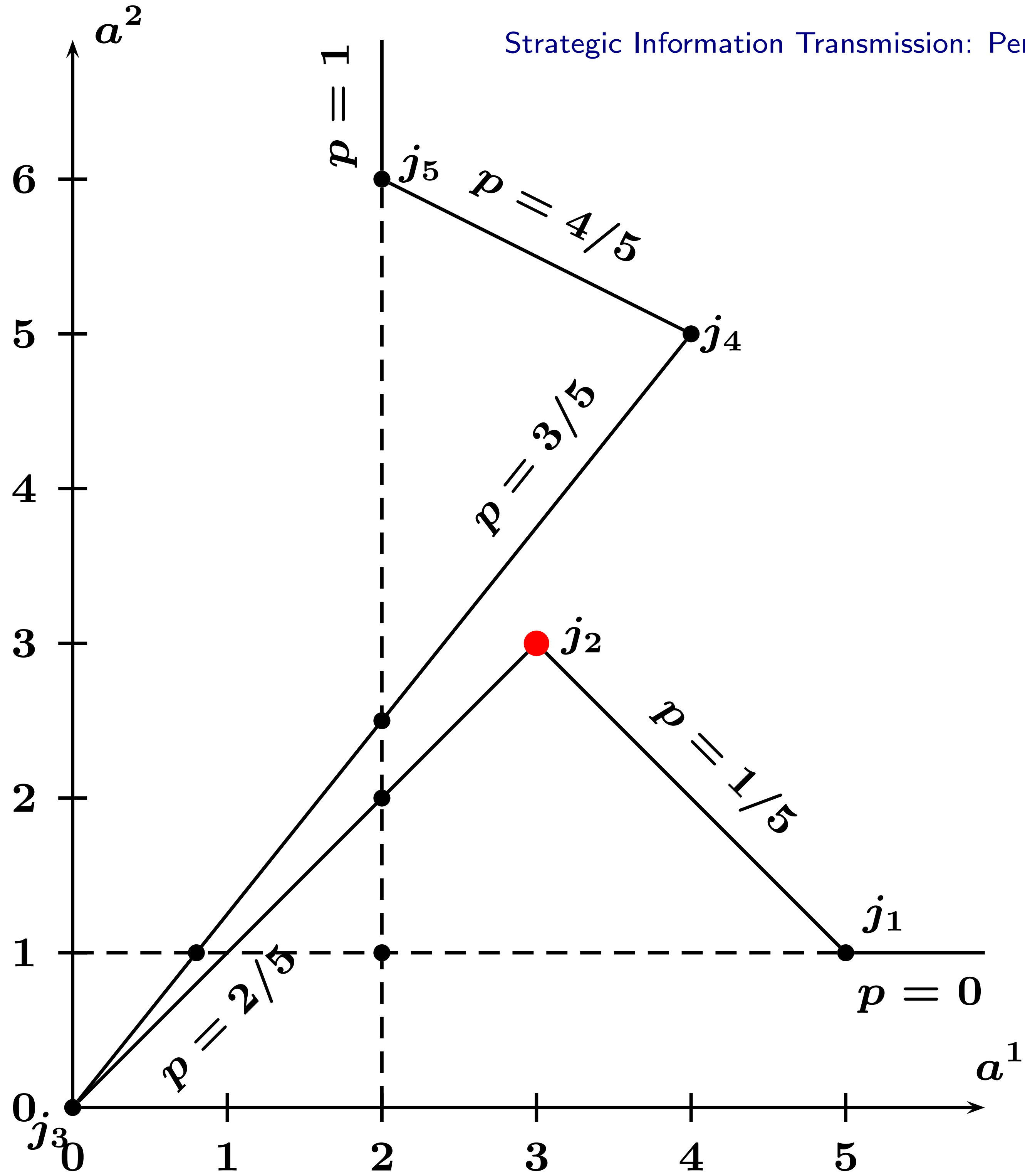
Information phase	Talking phase ($\boldsymbol{n \geq 1}$ rounds)	Action phase
Expert learns $\boldsymbol{k \in K}$	Both send $\boldsymbol{(m_t^1, m_t^2) \in M(k) \times M^2}$ $\boldsymbol{(t = 1, \ldots n)}$	DM chooses $\boldsymbol{j \in J}$

$\boldsymbol{\mathcal{E}_n(p)}$: Nash equilibrium payoffs of $\boldsymbol{\Gamma_n(p)}$

$\boldsymbol{\mathcal{E}_B(p) = \bigcup_{n \geq 1} \mathcal{E}_n(p)}$: NE payoffs of all multistage, bilateral persuasion games

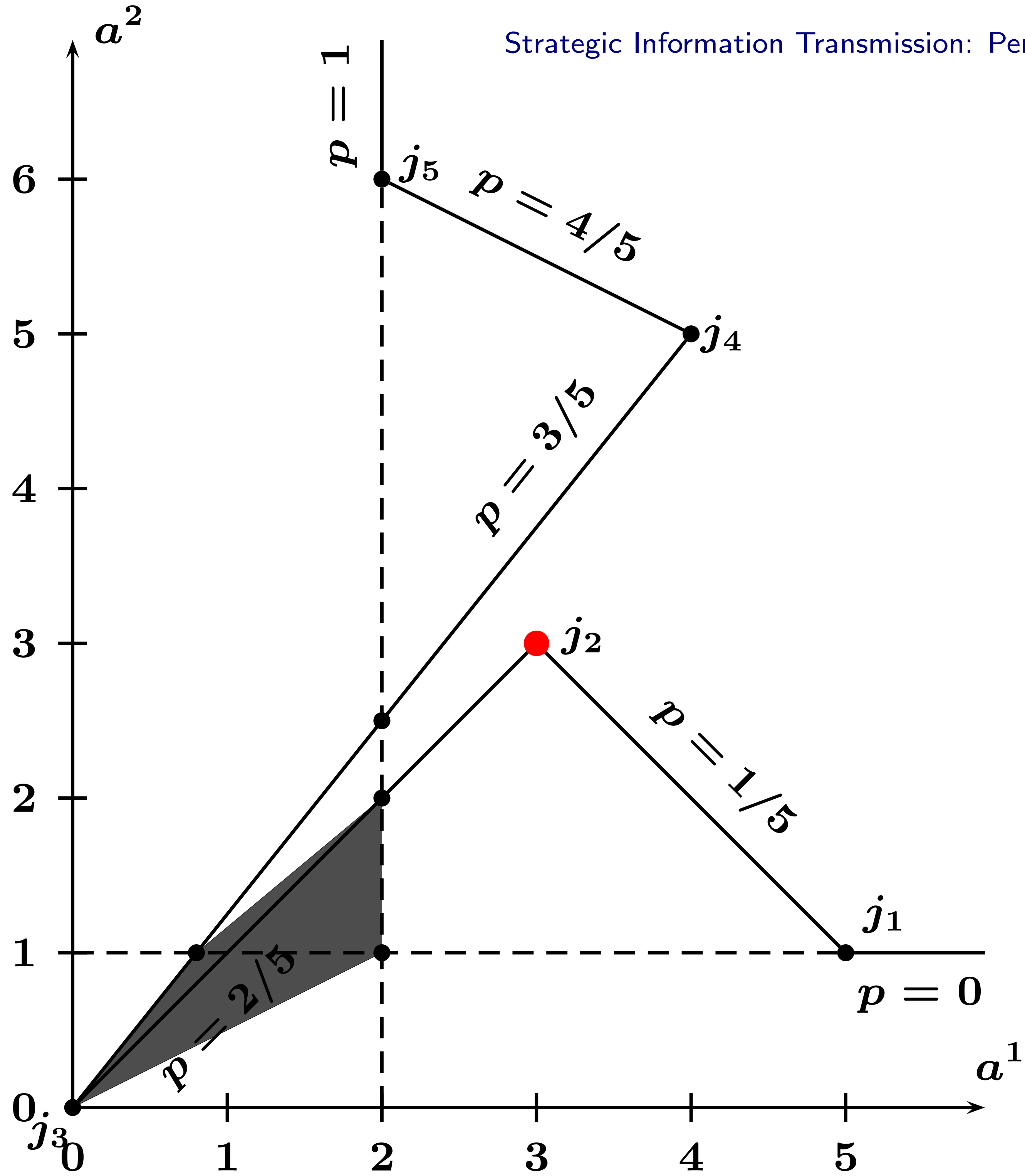
Theorem (Characterization of $\mathcal{E}_B(p)$) *Let* $p \in (0,1)$. *A payoff* (a, β) *is an equilibrium payoff of a multistage bilateral persuasion game* $\Gamma_n(p)$, *for some length* n, *if and only if* (a, β, p) *belongs to* di-co $(\mathrm{gr}\, \mathcal{E}^{++}) \cap$ INTIR, *the set of all points obtained by diconvexifying the set of all payoffs in* $\mathrm{gr}\, \mathcal{E}^{++}$ *that are interim individually rational for the expert:*

$$\mathcal{E}_B(p) = \{(a, \beta) \in \mathbb{R}^2 \times \mathbb{R} : (a, \beta, p) \in \text{di-co}\,(\mathrm{gr}\, \mathcal{E}^{++}) \cap \text{INTIR}\}.$$

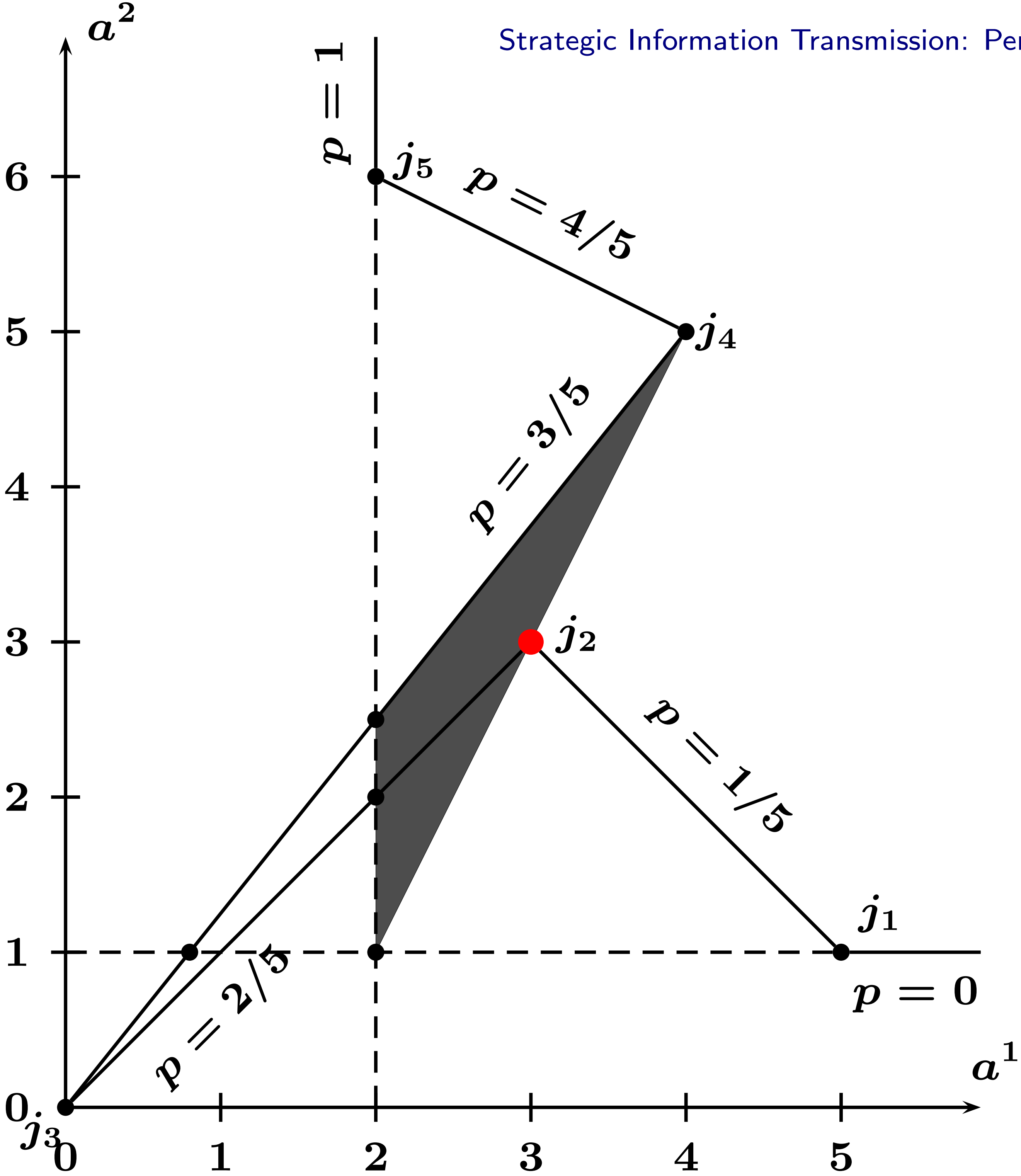


a^2
a^1
$p = 1$
$p = 4/5$
$p = 3/5$
$p = 2/5$
$p = 1/5$
$p = 0$
j_1
j_2
j_3
j_4
j_5
0
1
2
3
4
5
6

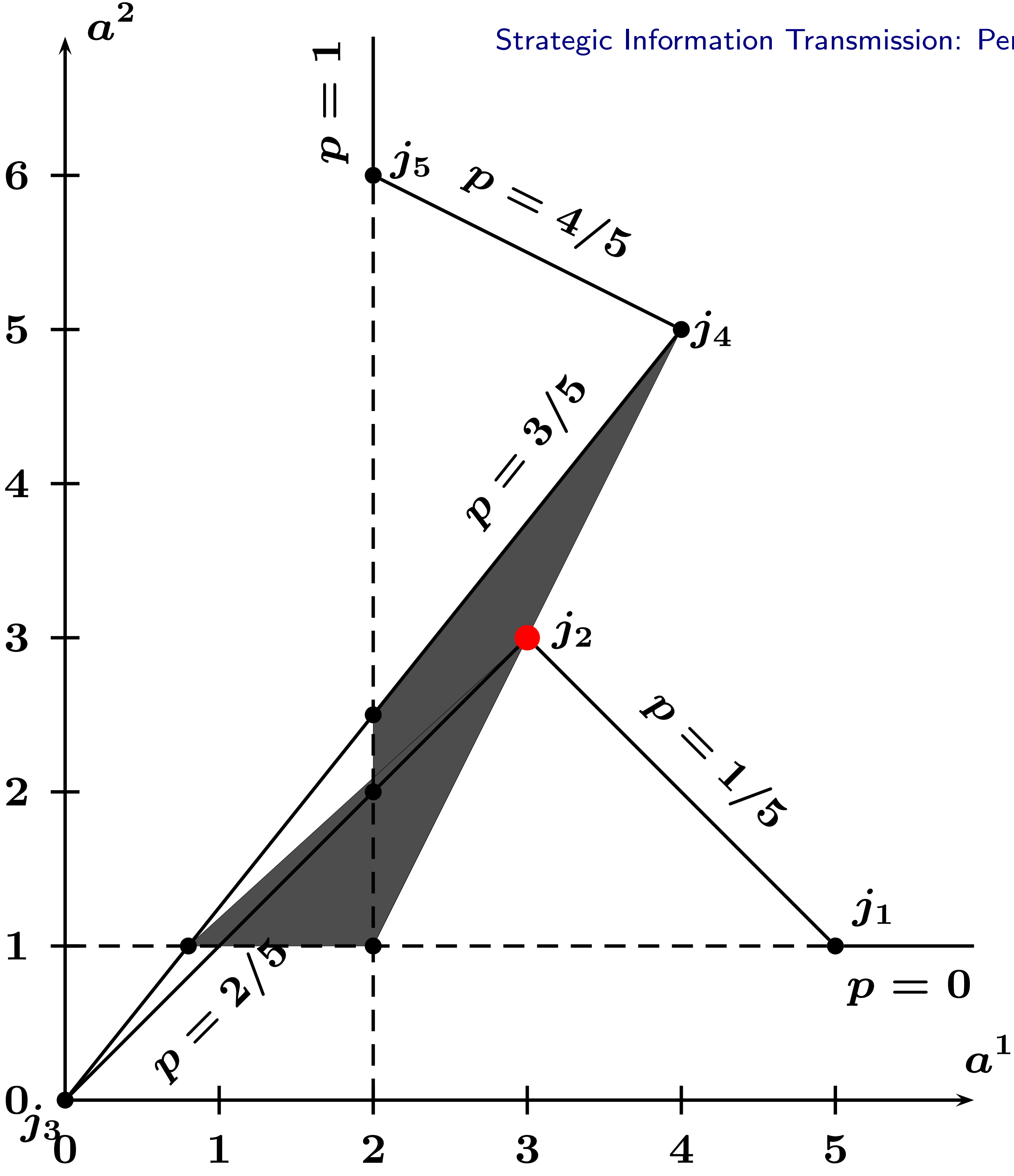
JCL-Signal
at $p = 1/2$



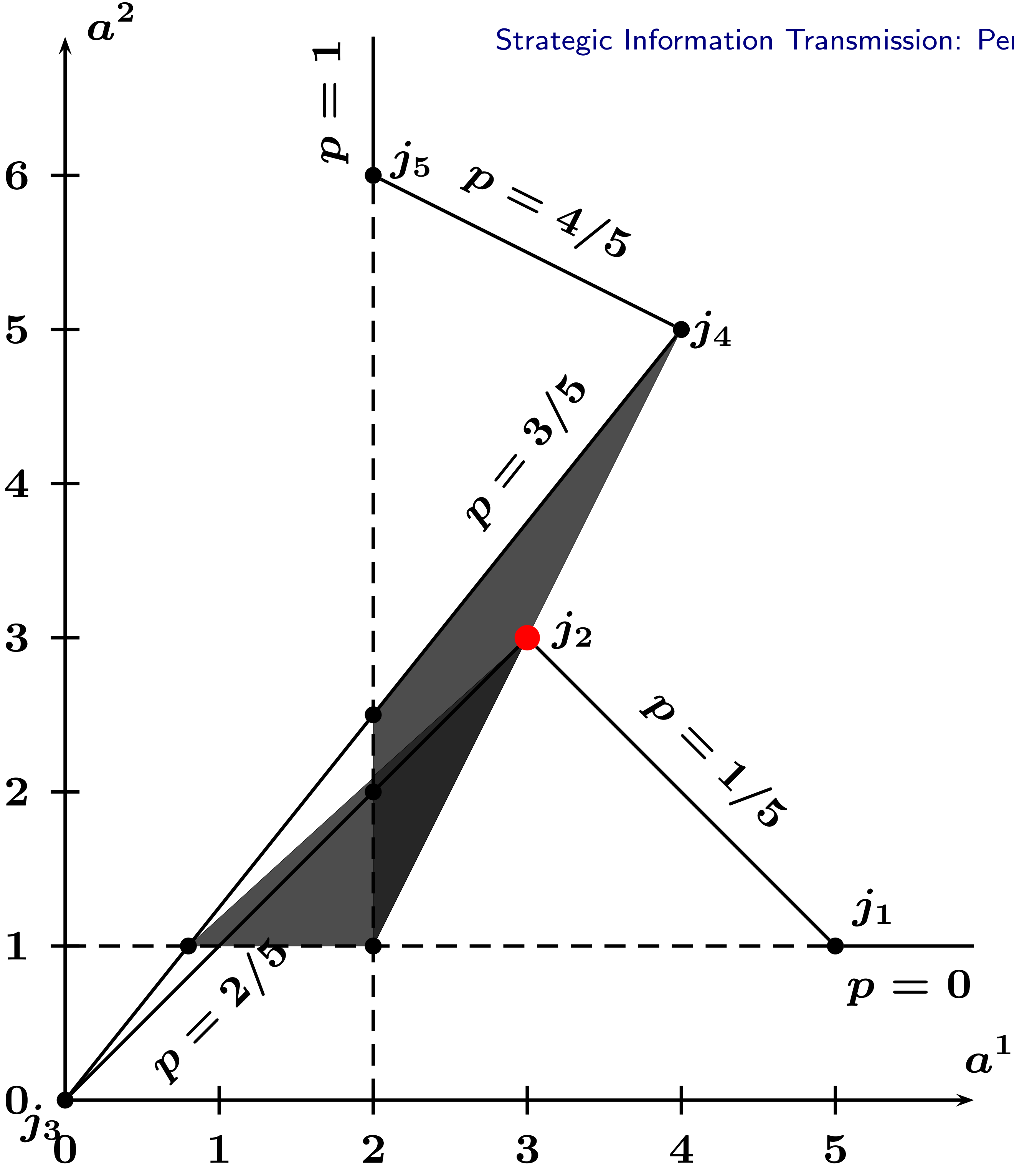
JCL-Signal
at $p = 3/4$



JCL-Signal
at $p = 1/4$



(subset of)
Signal-JCL-Signal
at $p = 1/2$



References

Crawford, V. P. and J. Sobel (1982): "Strategic Information Transmission," Econometrica, 50, 1431–1451.

Forges, F. and F. Koessler (2005): "Communication Equilibria with Partially Verifiable Types," Journal of Mathematical Economics, 41, 793–811.

——— (2007): "Long Persuasion Games," Journal of Economic Theory, forthcoming.

Green, J. R. and J.-J. Laffont (1986): "Partially Verifiable Information and Mechanism Design," Review of Economic Studies, 53, 447–456.

Grossman, S. J. (1981): "The Informational Role of Warranties and Private Disclosure about Product Quality," Journal of Law and Economics, 24, 461–483.

Milgrom, P. (1981): "Good News and Bad News: Representation Theorems and Applications," Bell Journal of Economics, 12, 380–391.

Milgrom, P. and J. Roberts (1986): "Relying on the Information of Interested Parties," Rand Journal of Economics, 17, 18–32.

Schelling, T. (1960): The Strategy of Conflict, Harvard University Press.

Seidmann, D. J. and E. Winter (1997): "Strategic Information Transmission with Verifiable Messages," Econometrica, 65, 163–169.